APR 18 1924

VOLUME LIX NUMBER 2

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

EDITED BY

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MARCH 1924

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THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE MISSION BOOK COMPANY, Shanghai

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The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 12*s*. 6*d*. each; single copies, including postage, 4*s*. each.

For Japan and Korea: The Maruzen-Kabushiki-Kaisha, 11 to 16 Nihonbashi Tori Sanchome, Tokyo, Japan. Yearly subscriptions, including postage, Yen 13.65 each; single copies, including postage, Yen 1.68 each.

For China: The Mission Book Company, 13 North Szechuen Road. Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business Correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Ryerson Laboratory, University of Chicago, Chicago, Illinois.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the Act of March 3, 1897.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

PRINTED IN THE U.S.A.

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THE OPTICAL PARTS OF THE VICTORIA SPECTROGRAPH

By J. S. PLASKETT

ABSTRACT

The optical system originally planned for the Victoria spectrograph, owing to war and other conditions, *has had to be considerably modified*, and this paper gives the results of tests of the field curvatures of four different types of camera objectives with one, two, and three prisms. The original, medium-focus triplet gives beautiful definition and a field flat to less than 0.1 mm over the whole visible spectrum with one prism; but, with two and three prisms, it is strongly concave. This curious behavior is ascribed to the chromatic aberration of collimator and camera, which tends to make the field convex, being exactly neutralized by the normal concave field curvature of the camera with the dispersion of one prism, while with greater dispersion the concavity of field overcomes the chromatic effect. A shorter-focus triplet of nearly the same type behaves very similarly.

A new modified Petzval type, of aperture f3, designed by Dr. G. W. Moffitt, was obtained and tested, and gave fine definition and flat fields with two and three prisms, but a convex field with one prism. A modification of this to give flat field with one prism for low spectra of faint stars is also being obtained. For higher dispersion with two and three prisms for which the triplet is not suitable, the well-known *Ross wide field astronomical lens* has been adapted by Dr. Ross to spectroscopic work, and a specially computed example gives excellent definition and flat fields with two and three prisms, but again a strongly convex field with one prism. This similar behavior of the Moffitt and Ross lenses is again to be ascribed to the chromatic aberration of collimator and camera which the nearly flat fields of the two lenses practically compensate at the wider angular extent of two- and three-prism dispersion, but not with one prism. These two new lenses make the spectrograph complete with critical definition and flat field for all practicable and useful dispersions.

INTRODUCTION

This instrument, a successful attempt to combine the stability of the fixed-form type, now generally used in stellar spectrographs, with the flexibility of the universal type which permits the use of one, two, or three prisms with various focal lengths of camera at any

desired deviation, has already been fully described.[1] This description was, however, relatively incomplete so far as the optical parts were concerned, as neither the prisms nor camera lenses required for the necessary range of dispersions were then available. The completion of the complement of prisms and the addition of three new types of camera objectives seem to justify a further account of the optical parts of the spectrograph and of the results of the tests of the new lenses. This description does not profess to attack the subject from the standpoint of the lens designer, for which the writer freely confesses insufficient knowledge, but rather from that of the astronomer, who is interested more particularly in the resulting spectrum as regards dispersion, defining power, and curvature of the field than in the technical details of the lenses producing these results.

As the instrument was originally described, the only dispersing piece available was a 60° Hilger prism of O 118 Jena glass, of 63.5 mm effective aperture. This prism was kindly lent by Professor Chant, of the University of Toronto, to whom the Observatory is under great obligations, as otherwise the spectrograph would probably have remained idle for fifteen months. Although the optical equipment as originally planned has been considerably modified for reasons that will appear as the description proceeds, it is of advantage to give in Table I the focal lengths of the cameras first proposed, with the resulting dispersions.

TABLE I

DISPERSIONS ORIGINALLY PROPOSED

Material	Number of Prisms	Angle of Prisms	Transmission of Prisms	Deviation of Prisms at λ 4200	Angular Dispersion at H_γ	Linear Dispersion at H_γ			Resolving Power at λ 4200
						Focal Lengths of Cameras			
						381 mm	711 mm	965 mm	
O 118....	1	60°	.768	50° 0′	8″.51	63.6	34.1	25.1	Toronto prism
O 118....	1	63	.756	54 40	9.86	54.9	29.4	21.7	30450
Ordinary.	2	63	.603	109 20	19.72	27.4	14.7	10.8	60900
Flint.....	3	63	.503	164 0	29.58	18.3	9.8	7.2	91350

[1] *Astrophysical Journal*, **49**, 209, 1919; *Publications of Dominion Astrophysical Observatory*, **1** No. 1.

THE PRISMS

The material of the prisms, O 118 Jena glass, one of the ordinary flints, was selected principally on account of its high transmission in the violet, and consequent facility in recording H and K of calcium, with, at the same time, fairly high dispersion. Experience has shown the great advantage of this material over the denser O 102 previously used at Ottawa and elsewhere. Although three prisms of this glass, of 63° angle and of suitable sizes to take a 63.5 mm pencil, were ordered from the John A. Brashear Company in 1914, they were not available until 1920. The fine definition of the borrowed Hilger prism, with uncertainty when the Brashear prisms would be ready, led to obtaining a new prism from Hilger, of material of about the same optical constants, which was brought into use in August, 1919. This prism, corrected by Hilger's interferometer method, is of exquisite defining power, unequaled, I believe, in any stellar spectrograph, and consequently it was retained for one-prism work even after the Brashear prisms arrived. The second and third Brashear prisms are used in combination with the Hilger, when higher dispersions are required, and also give beautiful definition.

ORIGINAL OBJECTIVES

The collimator objective of 63.5 mm (2.5 in.) aperture and 1143 mm (45 in.) focal length and the two camera objectives of 76 mm (3 in.) aperture and 711 mm (28 in.) and 965 mm (38 in.) focal lengths, respectively, were supplied with the spectrograph, and are of the Hastings-Brashear triplet construction corrected for wave-length λ 4200. The internal adjacent surfaces of the components are of like radii, and this is a considerable advantage as it allows the use of a film of transparent watch oil between the components, reducing the loss by internal reflections by 15 per cent in each lens. Although the color curve of the collimator has not been directly determined, it is obvious, from the final results with different camera lenses, that there is considerable residual chromatic aberration which appears to amount to about a millimeter longer focus for wave-length at λ 4800 than at minimum λ 4200. Although this aberration has the disadvantage of producing convexity of

field when used with a flat field camera lens, as is shown by Figure 2, it has the advantage of compensating to some extent the normal concavity of field of most lenses.

In the particular combination which has been used in over 90 per cent of the work with the spectrograph, one prism and the triplet camera of 711 mm focus, giving a linear dispersion at Hγ of 29 A to the millimeter, we have been particularly fortunate in this compensation. As will be seen by the curve in Figure 1[1] for the 711 mm camera with one prism, the whole spectrum between λ 3800 and λ 6700 is flat to within about a tenth of a millimeter. That this remarkable flatness, inclined about 2° to the normal to the axis with the violet end of shorter focus, depends upon the compensation above referred to is evident on examining the field curves given by the same objective with two and three prisms. Here, for any particular wave-length, which is two and three times as far from the axis as with one prism, the concavity produced by the camera is too great to be compensated by the convexity of the color curve of the collimator, and the field is strongly concave. That even a much smaller change in dispersion will destroy this balance was shown by similar objectives at Ottawa giving a strongly concave field when used with one prism of the O 102 glass, which is 30 per cent more dispersive.

The triplet camera lens of 965 mm focus gives fields of practically the same form, but has not been used in stellar work. This has been chiefly on account of the advantages of a higher ratio of collimator to camera focus, and the desirability, if higher dispersion than given by the 711 mm camera was required, of obtaining it by adding one or two prisms and using a medium or short-focus camera. Again, the dispersion given by the 711 mm triplet is about the most suitable for one-prism work, and when with this is combined almost complete flatness of field with ideal defining

[1] It is perhaps desirable at this stage to explain the conventions used in representing the field curves in Figs. 1 and 2. In order to show definitely the form, the vertical scale is exaggerated ten times. The vertical separation of the cross-section lines represents 1 mm of focal distance, while the horizontal separation corresponds to 10 mm on the plate. The positions of the wave-lengths are indicated on each curve, while the small circles represent the observed positions of focus determined photographically by a simplified Hartmann extra-focal method.

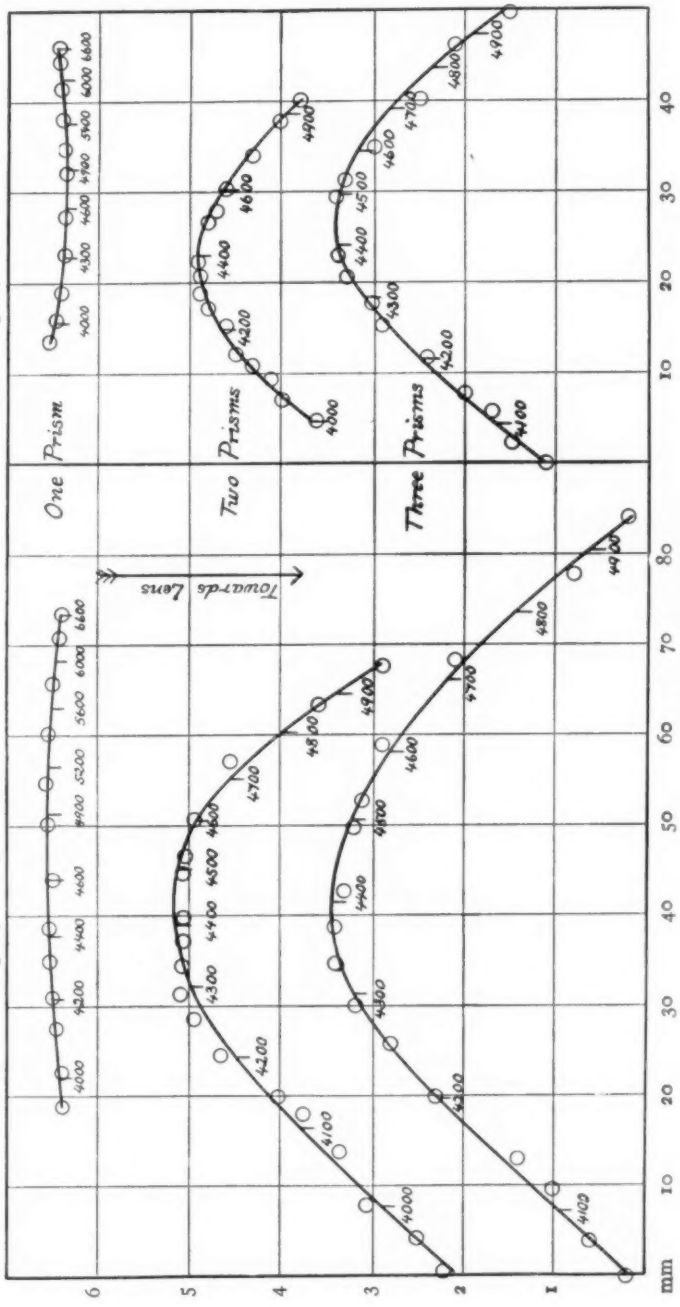


FIG. 1.—Field curves of triplets

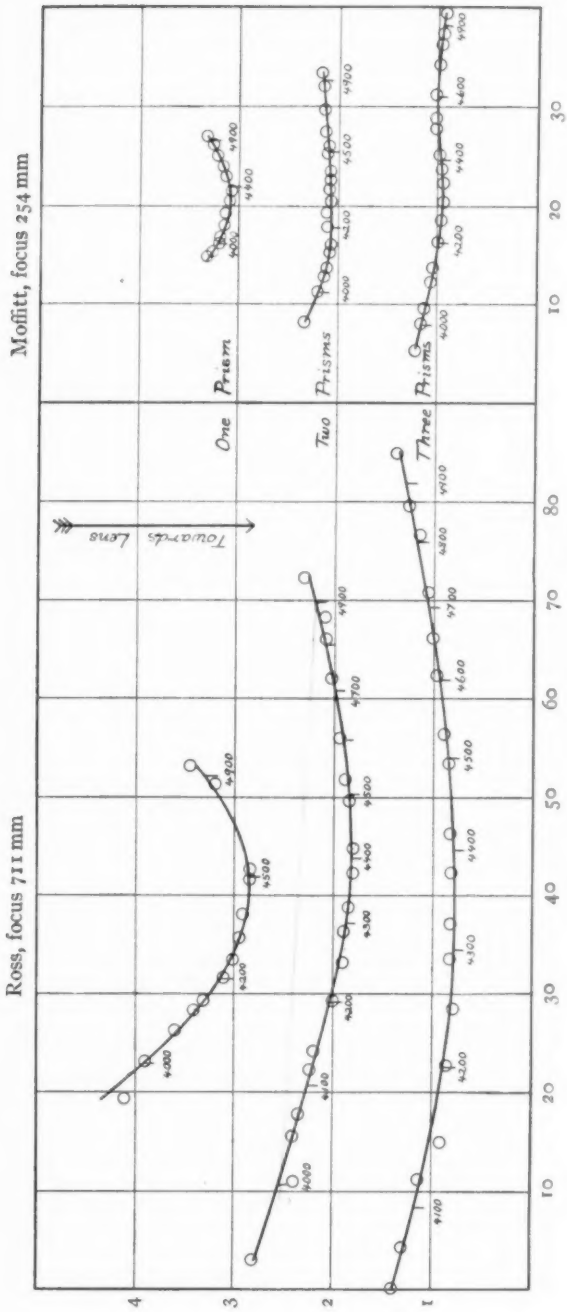


FIG. 2.—Field curves of new lenses

power over the whole spectrum, minimum losses by reflection and absorption, and practically constant focal settings for all temperatures, we may consider ourselves specially fortunate in having this happy combination available for the majority of our work.

CAMERA OBJECTIVES OF SHORTER FOCUS

As the exposure required under average observing conditions for stars about 7.5 mag. with the foregoing combination will exceed one hour, a smaller dispersion will be required for fainter stars, most readily obtained by the use of a camera of shorter focus. Considerable difficulty was, however, experienced in obtaining a satisfactory objective. A first trial by the Brashear Company in 1919 of a modified Cooke triplet with separated components of 76 mm (3 in.) aperture and 380 mm (15 in.) focus was unsuccessful, but a second attempt, using a modification of their regular triplet by omitting the internal contact feature for one component, was more fortunate. This gave one more radius to work with, and by diminishing the aperture ratio to f6, making the objective of 70 mm aperture and 420 mm focus, a very satisfactory objective was obtained, giving definition and flatness of field nearly equal to that of the 711 mm triplet. The linear dispersion is about 49 A to the millimeter at Hγ when used with one prism, and as is shown by the curve in Figure 1, the whole spectrum can be obtained in good focus on a flat plate. While the decreased dispersion cannot be expected to give as high an accuracy of measurement, it practically increases the observing capacity by one stellar magnitude. For stars with broad or diffuse lines, such as the O-type now being investigated, the loss in dispersion is partially compensated in the measurement by the consequent narrowing of the broad lines, and this objective has proved very useful.

For obtaining spectra of stars much fainter than the eighth magnitude in a reasonable time, an objective of still shorter focus is required. The difficulty of obtaining a satisfactory spectrographic objective of f6 will be increased many times when an aperture ratio two or three times as great is required, and the history of previous attempts, when critical definition is an essential, has not been very promising. It was not until some twelve months

ago, when in correspondence with Dr. F. E. Ross, of the Kodak Research Laboratory, concerning a suitable objective for use with three prisms, that I learned that Dr. G. W. Moffitt, of the same institution, had designed a modified Petzval type which could be used at f2. As a focal length of less than 225 mm could not be used without special adaptation of the spectrograph, an objective of 76 mm (3 in.) aperture and 254 mm (10 in.) focus was ordered from J. B. McDowell, successor to the Brashear Company. This lens, designed by Moffitt, was received last fall and immediately tested. The definition and resolving power are excellent at all dispersions, but the lens is too well corrected for field to give the best results with one-prism dispersion. The field curves of the Moffitt lens in Figure 2 show the effect of the color curves of the collimator and camera very clearly. A convexity of about two-tenths of a millimeter with one prism is reduced with two and three prisms to less than one-tenth. With a lens of such a large aperture ratio, f3, and such critical defining power as this lens possesses, a deviation of a tenth of a millimeter from the focus is at once apparent, and the field with one prism is therefore too curved for the best results with a flat plate. With two and three prisms, however, practically the whole extent of the photographic part of the spectrum is in focus to the twentieth of a millimeter, and the lens performs beautifully at these dispersions, 40 A and 26 A to the millimeter at Hγ. The Moffitt lens, with two and three prisms, has obvious applications in nebular spectra where the large aperture ratio will much shorten the exposures. It will also be useful where greater purity than is given by one prism is required in stellar work. For the same purity of spectrum, with two or three prisms a slit two or three times wider than with one prism can be used, and it may considerably shorten exposure time under most conditions of seeing. But considerable caution is required with slits wider than normal on account of asymmetrical illumination of the slit opening and consequent systematic displacement of the star lines. Although the present example has not fulfilled its main purpose of providing minimum linear dispersion for faint stars, there can be no doubt, as the main difficulty in camera objectives is to overcome concavity and not convexity of field, that a lens of this type can be easily designed to

give a flat field with one-prism dispersion. Dr. Moffitt is undertaking the computation of a camera objective of a ratio of f3, 76 mm (3 in.) aperture and 228 mm (9 in.) focal length to give a flat field with one prism, availing himself of the data obtained from the tests of the present lens. This is not yet ready, but I have no doubt it will meet our rather exacting requirements and will complete the equipment for low-dispersion spectra, 90 A to the millimeter, at Hγ, for use with faint objects.[1]

CAMERA OBJECTIVES FOR HIGH DISPERSION

When higher dispersion or purity than that given by the triplet camera objectives with one prism is required, the obvious means is to use two or three prisms. But the same triplet which gives such ideal definition and field with one prism or the modified shorter-focus triplet gives hopelessly concave fields with two or three prisms, so that not more than 200 A of the spectrum could be in satisfactory focus on a flat plate. This is very definitely shown by the field curves in Figure 1 and consequently, although the definition is excellent with the higher dispersion, the great curvature of field makes another type of objective necessary.

Although an objective of the Hartmann-Zeiss "Chromat" type of the same material as the prisms would, as already shown,[2] probably produce the requisite flatness of field, the required plate inclination, about 16°, is rather objectionable, and an achromatic combination which would give the focal plane nearly normal to the axis would be preferable. The production of the Ross astronomical photographic objective, which gives a remarkably flat field of about 15° with freedom from sensible coma, at f10, seemed to point the way toward meeting the requirements. A test of a sample objective of 3 inches aperture and 30 inches focus made for Dr. Schlesinger gave very promising results, and consequently an objective of this type of 76 mm (3 in.) aperture and 711 mm (28 in.) focal length, made of this particular focus to fit the same camera as the triplet, was ordered from J. B. McDowell and specially computed by Dr. Ross. This Ross objective is a quadruplet, four separated elements of nearly symmetrical construction, appears to be quite

[1] This new objective has been completed and gives a field flat within 0.03 mm.

[2] *Astrophysical Journal*, **29**, 290, 1909.

sensitive to changes in radius or separation, and, according to McDowell, is rather difficult to figure. We are much indebted to Dr. Ross for the computation and to Mr. McDowell for the special care in figuring. The objective gives very similar field curves to the Moffitt, and it seems to be a fair deduction that they each give normally a flat field, and that the convexity, most marked with one prism, is due almost wholly to the chromatic aberration of the collimator.[1] Its marked advantage over the triplet of the same focus for dispersions of two and three prisms is distinctly shown by a comparison of Figures 1 and 2. With either two or three prisms it is readily seen from Figure 2 that a flat plate can be so placed that it is at no point between λ 4100 and λ 4900 more distant than two-tenths of a millimeter from the focus. If the limits of wave-length be restricted to the region between λ 4150 and λ 4650, the deviation from focus will not exceed one-tenth of a millimeter, one part in seven thousand of the focal length, perfect definition. Indeed, no want of definition can be detected on a properly focused plate over the wider limits of 800 A. If further refinement and complete correspondence with the focal curve is required along the whole length, it is very easy to bend the plates to the exact radius of curvature, about 100 and 150 cm for two and three prisms, respectively. In the book-form holders used, a strip about a millimeter thick at each end of the plate and a slight increase of the pressure on the center produces the necessary curvature without difficulty or without permanent alteration of the plateholders. The objective may hence be considered practically perfect for the purpose for which it was designed—two- and three-prism dispersion. Like the Moffitt, it cannot be successfully used with one prism, but this is no drawback when the Brashear triplet, of the same focus, is ideal for this dispersion. We can well afford to confine the use of the Ross lens to the higher dispersions.

The completion of the Moffitt lens of 228 mm focus for the lowest dispersion with one prism, as can be seen from Table II, will make

[1] Both Dr. Ross and Dr. Moffitt have kindly suggested that the collimator is not responsible for all the chromatic effect, but that the resultant field in every case is due to the balance between the chromatic aberration of collimator and camera and the normal curvature of field of the camera.

the spectrograph complete for all dispersions likely to be required. A comparison of Table II, which contains only those dispersions for which the optical parts will give good field and definition, with Table I will show how the original plans for the spectrograph have been finally worked out.

TABLE II

DISPERSIONS FINALLY AVAILABLE

NUMBER OF PRISMS	MAKES OF PRISMS USED	LINEAR DISPERSION A PER MM AT H_γ				
		Moffitt Lenses		Brashear Triplets		Ross Lens
		228 mm	254 mm	420 mm	711 mm	711 mm
1.....	Hilger	90.0	.....	49.3	29.3	.....
2.....	I Hilger; II Brashear	.....	39.5	.....	.....	14.5
3.....	I Hilger; II, III Brashear	.....	25.9	.....	.....	9.6

When it is remembered that at each of these dispersions we have practically perfect definition and field over the whole spectrum that can be photographed on the plate, that the changes from one dispersion or deviation to another can be readily and quickly made, and the required settings reproduced at will, and that the spectrograph for any of these dispersions is practically as stable as a single-form instrument, the success of the reversion to the universal form of stellar spectrograph seems confirmed.

In conclusion, I would like to express my appreciation to each of the designers who have so freely given of their efforts toward the perfection of the objectives. Especially are my thanks due to Mr. McDowell for his care and great skill in the figuring of these lenses and for the spirit he has shown in never being satisfied with any of his work until it could no longer be improved.

DOMINION ASTROPHYSICAL OBSERVATORY
VICTORIA, B.C.
July 21, 1923

THE VACUUM SPARK SPECTRUM OF CALCIUM[1]

By J. A. ANDERSON

ABSTRACT

Vacuum spark spectrum of calcium λ 2000–λ 6600.—Calcium electrodes 6 millimeters in diameter with a gap of 2 millimeters were used in a glass vacuum chamber. Vacuum sparks were produced by the discharge of a 1.0 microfarad condenser charged to a potential of 20,000 volts. The spectrum was photographed with a 1-meter focus, 4-inch concave grating, and the longest exposure required was 400 sparks. The probable error of the wave-length of good lines is about ±0.02 A. The chief impurities were *Mg*, *Sr*, *O*, and *N*. Table I contains 850 lines which are ascribed to calcium, and in Table II are collected 46 of these, which represent all the lines observed which have previously been listed as calcium lines. It is likely that many of the new lines are due to Ca^{++}.

Specification of spark circuits.—It is suggested that spark circuits should be specified by the maximum value of the current, the maximum value of the potential difference between the terminals, and the frequency of oscillation, or by data from which these quantities may be readily obtained. The subject is discussed briefly and a few numerical examples are given. In the present investigation the maximum value of the current was about 10,000 amperes, the maximum potential difference 20,000 volts, and the frequency 87,000 cycles.

Intensity scale for spectra.—An attempt is made to give intensities of lines on a logarithmic scale to base 2, that is, a line of intensity m is 2^n times as bright as a line of intensity $m-n$. The method of exposure ratios is employed.

Direct wave-length reader for grating spectra.—A revolution counter provided with a small circle to read fractions of a revolution is geared to the head of the measuring machine, the gear ratio being so chosen that one revolution of the counter circle shall be very nearly equal to 1 A on the plate. Approximate wave-lengths can thus be read directly. Corrections are derived from similar readings on the comparison iron arc spectrum, and, as they are small, they are very easy to apply. Tests show no appreciable loss in accuracy and a very great saving in the time required to measure and reduce a spectrogram.

INTRODUCTION

1. About four years ago a glass plate condenser having a capacity of about one microfarad, and capable of being charged to a potential of 20,000 volts, was constructed in this laboratory. While employed principally in the electrical explosion of wires,[2] it has at various times been used to produce powerful sparks in air, also for vacuum-tube discharges and sparks between metal terminals in a high vacuum. The vacuum spark spectrum of a number of elements has been photographed, and it is the purpose of the present paper to give an account of the spectrum of calcium.

[1] *Contributions from the Mount Wilson Observatory*, No. 269.

[2] *Astrophysical Journal*, **51**, 37, 1920.

The vacuum spark has been used by Millikan and his co-workers for a number of years in the study of the extremely short wavelengths. Recently Sawyer and Paton[1] have published an account of the vacuum spark spectrum of silicon, and Shallenberger[2] has studied the spectrum of aluminum. These investigations show that in the ordinary photographic region the vacuum spark exhibits many lines not given by the arc, or by the spark in air. In the present investigation of the spectra nine elements have been photographed, all of which show some lines not given by the ordinary arc or spark. There is some indication that the number of lines may be a function of the atomic weight of the element. Carbon, aluminum, and magnesium, elements of low atomic weight, show relatively few new lines; while calcium, copper, and zinc show many, and cadmium and lead, elements of high atomic weight, each give more than a thousand lines not heretofore recorded.

2. The character of the vacuum spark and its spectrum will undoubtedly be found to depend upon a number of factors, of which the following may be the most important:

a) The pressure of the residual gas
b) The size and separation of the electrodes
c) The instantaneous value of the current passing between the electrodes
d) The voltage between terminals when the spark begins

It is to be regretted that in most accounts of work with spark and vacuum-tube spectra hitherto published, the authors have not given sufficient data for estimating these factors accurately or even approximately. This applies especially to the electrical quantities. The following considerations will make this clear: Let AB (Fig. 1) represent a high voltage transformer, and let the primary voltage be V, the secondary voltage $v=nV$, so that n is the ratio of transformation. If the primary current is I, the secondary current will be, approximately, $i=I/n$. It is well known from the theory of the transformer that these relations are only rough first approximations, but they will be sufficiently accurate for our purpose.

[1] *Ibid.*, **57**, 279, 1923.

[2] *Physical Review*, **19**, 398, 1922.

For simplicity, let us assume that S is a standard sphere gap adjusted to the voltage v. If the capacity of the condenser is C (farads), the current in it will be

$$i' = 2\pi N C v' \tag{1}$$

if $v' \lesssim v$, so that no spark passes across S. It follows that for i' approximately equal to i, as it will be in general, v' can only reach the value v if

$$C \lesssim \frac{i}{2\pi N v}. \tag{2}$$

As a numerical example, let us take $C = \frac{1}{100}$ microfarad, which is about the capacity of a good-sized Leyden jar; let $v = 50{,}000$ volts, $N = 60$ cycles, and we have $i = 0.01885$ amperes. The size of transformer required is given in watts by the product of i and v, which is in this case about 950; or, in other words, to charge a Leyden jar of the given capacity to 50,000 volts on 60-cycle current requires a transformer of at least 1 kw capacity. If the transformer capacity is less than that just given, the potential across the condenser will be less than 50,000 volts. From equation (1) it follows that the product of i and v, that is, the capacity or size of the transformer, will vary as the square of the voltage, and hence to charge the condenser we have just considered to 100,000 volts would require a 4 kw transformer.

From the foregoing considerations it follows that to know even approximately the voltage across the condenser terminals, when an ordinary alternating current is used, we must be given the size of the transformer, its secondary voltage, and the capacity of the condenser.

When an induction coil is used instead of a high-voltage transformer, it is more difficult to estimate what voltage may be effective. A statement such as "a 12-inch induction coil was used with a 1-quart Leyden jar in parallel with the spark gap" gives very little information except that the voltage across the Leyden jar was probably of the order of only a few thousand volts.

Since at 50,000 volts and 60 cycles a condenser of 0.01 microfarad capacity requires at least a 10 kw transformer, and, of course, a 1.0 microfarad condenser a 1000 kw transformer, it follows that

if a large condenser is to be used in an ordinary laboratory, it can be charged only by some source of unidirectional current, the most convenient being a high-voltage transformer with a suitable rectifying device. The transformer must furnish a current somewhat larger than the normal leak of the fully charged condenser, but that is practically the only requirement.

3. To find the instantaneous current in the discharge circuit we must know the capacity of the condenser, its voltage, and the inductance of the circuit; in lieu of the latter, which is difficult to measure directly, it is better to give the frequency of the oscillations in the discharge, which can be measured by merely photographing the spark with a rotating mirror camera. Let N_c be this frequency, the capacity and the voltage being respectively C and v as before. The maximum value of the current, assuming no damping, is

$$I_c = 2\pi N_c C v, \tag{3}$$

and unless the damping is extremely large, this will be very nearly the maximum value of the current during the first half-cycle. Applying this to the 1.0 microfarad condenser used in the present work, where

$$N_c = 87{,}000, \text{ and } v = 20{,}000,$$

we have

$$I_c = 10{,}927 \text{ amperes.}$$

If the damping be made critical, which in the present case requires the insertion into the circuit of a resistance of 2.5 ohms, the maximum value of the current falls to $1/e$ of this value, e being the base of the Napierian system of logarithms. If the inductance L is known, the formula (3) can be put in the following well-known simple form

$$I_c = v\sqrt{\frac{C}{L}}, \tag{4}$$

which shows that the voltage is more important than the capacity, since the latter appears under the radical.

4. It may be of interest to point out that the currents involved in ordinary condensed discharges, such as are frequently used with

vacuum tubes, are in reality very large. Let us assume a condenser of 0.001 microfarad capacity in parallel with the vacuum tube, and a series spark gap of a few millimeters in length, so that the voltage will be of the order of 5,000 volts. We may take the inductance of the ordinary discharge circuit as 0.002 millihenry. Using formula (4) with

$$v = 5{,}000, \qquad C = 10^{-9}, \qquad \text{and} \qquad L = 2 \times 10^{-6},$$

we obtain

$$I_e = 110 \text{ amperes}$$

approximately, as the maximum value of current. Since without the condenser the current would most likely be of the order of a few milliamperes, it is not surprising that the spectra given by the two arrangements should be radically different from each other.

EXPERIMENTAL

5. In the present work the sparks are produced by the 1.0 microfarad glass plate condenser, which is charged through a synchronous rectifier from the secondary of a 26,000-volt transformer rated at 0.5 kw. A full charging (to 20,000 volts as measured by a 6-inch sphere gap) requires from 4 to 5 seconds. The rectified current at the beginning of the charging is about 20 milliamperes, but it decreases rapidly as the condenser charges up, becoming very small at the end of 4 seconds. The condenser is discharged through the vacuum spark gap by closing a 3-cm gap which is in series with the former in the discharge circuit. The current in the vacuum spark gap is hence always the same, the maximum value in the first half-cycle being of the order of 10,000 amperes. The potential difference between the vacuum spark terminals, however, will depend upon the voltage drop across the external spark gap. It can never exceed 20,000 volts, and, in general, will be considerably less.

6. The vacuum chamber is shown in Figure 2. When in use the bulb *A* is apt to become heavily coated with metal evaporated from the electrodes *FF*, but the quartz window remains quite clean. By keeping an electric fan blowing across the apparatus the seals at *CC* and *EE* are kept sufficiently cool, even for a long run.

At first a single-stage mercury diffusion pump, backed by an oil pump, was tried for maintaining the vacuum, but without a

trap cooled by liquid air this was found unsatisfactory. A new Cenco high-vacuum pump was then substituted, and has given good results.

The appearance of the spark is used to tell when the vacuum is sufficient. At pressures from one atmosphere down to 10 or 15 centimeters of mercury, these powerful sparks look very much like the ordinary condensed spark, except that they are somewhat more expanded in the equatorial plane. As the pressure is reduced to a few millimeters, the spark expands enormously and loses rapidly in brightness in the vicinity of the electrodes, so that when the pressure is in the region 1.0 to 0.1 mm the light is apparently very little brighter between the electrodes than at other places in the

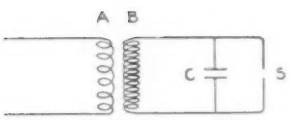


FIG. 1

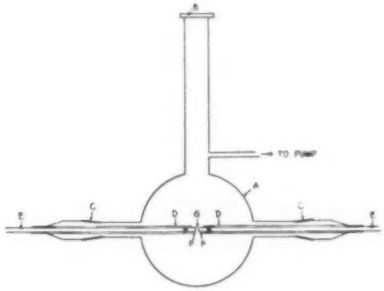


FIG. 2

bulb. As the pressure is further lowered, a stage is finally reached when an exceedingly bright layer appears on each electrode; at still lower pressures this layer grows until it appears to fill the gap between the electrodes completely. At this stage there is only a relatively feeble glow through the rest of the bulb. There is no further change in the appearance with diminishing pressure until the latter becomes so low that no discharge whatever can be made to pass. It is this extremely brilliant light filling the space between the ends of the electrodes that is called the vacuum spark, the spectrum of which has been studied.

The procedure followed, after a new pair of electrodes has been inserted, is to wait about 10 minutes for the pump to bring the pressure to about the right value and then pass from 30 to 50 sparks to heat the electrodes and drive off adsorbed gases, after which, as a

rule, the sparks assume the proper appearance, and an exposure can be started. With the metals calcium, zinc, cadmium, and aluminum it was found that when the electrodes became hot and sparks were passed at the rate of one every 5 seconds, the pressure would fall so low that the gap became non-conducting. A wait of a few seconds would, in such a case, allow sufficient gas to diffuse back from the pump to permit the discharge to pass again. Such an effect was expected with calcium, since it is well known that the hot metal will absorb all the common gases, but its occurrence with the other metals was a little surprising. It did not occur with magnesium, iron, copper, or lead.

7. The spectrum was photographed with a 1-meter focus concave grating of the 4-inch size, used in a fixed Rowland mounting. Eastman portrait films 2.5 inches by 15 inches (6.3 cm by 37.8 cm) covered the region from λ 2000 in the first-order spectrum, to λ 4080 in the second order, and recorded the first order as far as λ 5200 A. The same films, sensitized by Wallace's formula, were employed for the region λ 5000 to λ 7000 A.

The definition of this grating is very good, so that on the photograph two sharp lines separated by as much as 0.25 A are easily resolved in the first order, the corresponding separation for the second order being about 0.12 A.

In order to facilitate the estimation of the intensity of the lines, a series of exposures was made on the same film, with all conditions as nearly the same as possible except the exposure times. The latter were made to form, as nearly as possible, a geometrical progression. A few trials showed that 400 sparks gave a very full exposure; accordingly, the exposures were 400, 100, 25, 6, and, in a few cases, 2 sparks. An iron-arc spectrum was also impressed on each film. To assign intensities, a line which is just comfortably visible on the 6-spark exposure is given the number 6; one just visible on the 25-spark exposure is given the number 4; one just visible on the 100-spark strip is called 2, and one just seen on the 400-spark strip is called 0. A line of intensity 1 is easy to see on the 400, but is not measurable on the 100-spark strip, etc. If the sparks were all of the same intensity, and the reciprocity law held exactly, this procedure would give a rational intensity scale, like that of

stellar magnitudes, except that here we would use the ratio 2:1 instead of the astronomical ratio 2.5+:1. But on account of the variation of the intensity of the sparks, and perhaps also on account of the inexactness of the photographic law, it was found that only the strips 400, 100, and 25 could be trusted. A number of exposures each to 6 sparks showed variations in the intensity amounting to nearly one unit of the scale. In spite of this the 6-spark exposure was found very useful in assigning intensities to the stronger lines.

8. The scale of the spectrograms is about 16.8 A per mm in the first order. The measuring machine used has a screw of 0.5 mm pitch, and a head graduated to read to 0.001 mm. When all the readings are made in millimeters, it is necessary to use a calculating machine in order to apply the factor of approximately 16.8 to reduce the readings to angstroms, which, in case of a spectrum of hundreds of lines, is a rather tedious task. In order to simplify this part of the work, a revolution counter was geared to the head of the machine, the gears being so chosen that the counter-shaft makes 8.4 revolutions to one of the screw. A graduated head, divided into 100 parts, is mounted on the shaft. By setting the cross-wire on a known line and adjusting the counter to read its wave-length, approximate wave-lengths may be read to 0.01 A directly. Since the scale varies somewhat along the spectrogram, wave-lengths read this way are not correct, but by taking readings on the known lines of the iron-arc comparison spectrum which appears on each plate, the corrections are easily ascertained, and, as they never exceed 5 or 6 A, are easily applied. To transfer the corrections obtained from the iron-arc spectrum to the unknown spectrum, it is necessary to know the true wave-length of at least one line of the latter. In all the spectra so far studied, several well-known lines appear, so that this condition is fulfilled.

To determine whether there is any loss in accuracy due to the use of the counter, about a hundred lines well distributed through the iron-arc spectrum were selected, and their wave-lengths determined, first, by reading only the counter and deriving the corrections from ten of the lines used as standards; second, by taking readings directly from the head of the machine, and reducing in the usual way, using five lines as standards. The means of three sets

of readings were used in each case. In both cases the probable error of the wave-length of an "unknown" line was just a little under ± 0.02 A, showing that any loss in accuracy due to the counter is very slight indeed. The time required for the work, using the two methods, was approximately 1:5, in favor of the counter.

RESULTS

9. The first and second columns of Table I give the wave-lengths and intensities of the lines which are provisionally assigned to the calcium vacuum spark. Known lines of impurities have been eliminated as far as possible. The third and fourth columns give the calcium spark lines found by Eder and Valenta. Their wave-lengths have been rounded off to the nearest hundredth of an angstrom, and reduced to the international scale by applying Kayser's corrections. Eder and Valenta's table was selected for this comparison because it gives many lines in addition to those found by other spectroscopists. If we except the well-known calcium lines, collected for convenience in Table II, it is astonishing how few coincidences there are between Eder and Valenta's lines and those of the vacuum spark.

The new lines given in Table I must be due to one or more of the following sources:

a) The constituents of the electrode points *FF*
b) The metal (chiefly iron) composing the electrode leads *EE*
c) The glass shields *DD*
d) The residual gas in the bulb *A*

Lines due to sources *b*, *c*, and *d* should appear in the spectra of all metals, since only the electrode points *FF* are changed when another metal is to be studied. A comparison of the spectra of calcium, cadmium, and carbon reveals no lines due to (*b*) or (*c*) with the exception of D lines of sodium, which may come from the glass. The D lines, however, are relatively very weak, which makes it probable that their origin is (*a*). The comparison reveals surprisingly few lines which can be attributed to the residual gas, which at first led the writer to believe that the spectra are almost free from lines due to this source. A careful comparison of the spectra of calcium, cadmium, and carbon showed fewer than

TABLE I*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
.....	.....	2081.47	1	.....	2447.23..	4	.....	.....	.....
.....	.....	2086.58	1	.....	2470.28..	1	.....	.....	.....
2094.51..	0	.....	.....	.....	2472.52..	1	.....	.....	.....
.....	.....	2099.81	1	.....	2493.00..	7	.....	.....	.....
.....	.....	2103.41	1	.....	2497.67..	5	.....	.....	.....
.....	.....	2112.95	1	.....	2517.03..	6	.....	.....	.....
2113.79..	0	.....	.....	.....	2522.06..	2	.....	.....	.....
2122.51..	1	2122.9	1/2	.....	2528.83..	0	.....	.....	.....
.....	.....	2131.1	1/2	.....	2530.57..	6	.....	.....	.....
.....	.....	2132.9	1/2	.....	2541.49..	6	.....	.....	.....
2139.90..	1	.....	.....	.....	2547.88..	6	.....	.....	.....
2152.07..	1	.....	.....	.....	2553.04..	1	.....	.....	.....
2171.20..	0	.....	.....	.....	2556.45..	4	.....	.....	.....
2182.99..	1	.....	.....	.....	2565.48..	4	.....	.....	.....
.....	.....	2197.96	2	.....	.....	.....	2568.00	3	.....
.....	.....	2200.4	1/2*h*	.....	2574.32..	1	.....	.....	.....
2207.13..	1	.....	.....	.....	.....	.....	2575.13	3	.....
.....	.....	2208.88	2	.....	2587.09..	3	.....	.....	.....
2209.45..	0	.....	.....	.....	2590.34..	2	.....	.....	.....
2217.26..	0	.....	.....	.....	2609.57..	3	.....	.....	.....
2249.28..	0	.....	.....	.....	2620.82..	6	.....	.....	.....
2252.65..	2	.....	.....	.....	2624.11..	4	.....	.....	.....
2256.33..	0	.....	.....	.....	2630.00..	1	.....	.....	.....
.....	0	2259.4	1/2*h*	.....	2630.49..	1	.....	.....	.....
2267.19..	2	.....	.....	.....	2632.69..	1	.....	.....	.....
.....	.....	2275.37	2	.....	2634.17..	6	.....	.....	.....
2276.54..	2	.....	.....	.....	2642.74..	3	.....	.....	.....
2289.41..	3	.....	.....	.....	2653.32..	2	.....	.....	.....
.....	.....	2290.02	1	.....	2655.41..	1	.....	.....	.....
2306.97..	1	.....	.....	.....	.....	.....	2660.43	4	.....
.....	.....	2309.12	1	.....	2672.04..	1*h*	.....	.....	.....
2310.87..	0	.....	.....	.....	2680.12..	2	.....	.....	.....
2312.08..	3	.....	.....	.....	2684.38..	5	.....	.....	.....
.....	.....	2312.94	1	.....	2686.73..	3	.....	.....	.....
2316.27..	2	.....	.....	.....	2687.78..	8	.....	.....	.....
2320.26..	1	.....	.....	.....	2689.21..	1	.....	.....	.....
2322.91..	3	.....	.....	.....	2691.31..	0	.....	.....	.....
2334.13..	2	.....	.....	.....	2699.63..	2	.....	.....	.....
2339.88..	1*h*	.....	.....	.....	2704.87..	6	.....	.....	.....
2343.04..	2	.....	.....	.....	2710.60..	4	.....	.....	.....
2346.05..	1*h*	.....	.....	.....	2715.08..	3	.....	.....	.....
2348.91..	1	.....	.....	.....	2718.38..	5	.....	.....	.....
2351.40..	1*h*	.....	.....	.....	2732.56..	4	.....	.....	.....
2354.31..	0*h*	.....	.....	.....	2736.54..	6	.....	.....	.....
.....	.....	2373.16	2	.....	2754.18..	2	.....	.....	.....
2378.59..	2	.....	.....	.....	2756.45..	0	.....	.....	.....
2393.20..	3	.....	.....	.....	2759.48..	1	.....	.....	.....
.....	.....	2398.65	2*h*	.....	2762.12..	1	.....	.....	.....
2431.08..	1	.....	.....	.....	2766.28..	1	.....	.....	.....
2442.54..	1	.....	.....	.....	2771.27..	4	.....	.....	.....

* *h* = hazy.
H = very hazy.
s = sharp.
g? = ghost?

hr = hazy toward the red side.
hv = hazy toward the violet side.
d = double.

r = component on red side.
v = component on violet side.
R = reversed.

TABLE I—*Continued*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
2773.86	1				2910.31	4			
2777.30	0				2911.65	1			
2778.20	0				2913.68	2			
2782.47	1				2914.75	2			
2783.99	1				2915.12	1			
2785.91	0				2918.42	2*v*			
2787.68	0				2919.14	2			
2789.52	0				2920.46	3			
2791.63	6				2924.33	8			
2796.29	1				2928.22	1*h*			
2797.15	1						2928.81	4*h*	*Mg*?
2799.16	1*h*				2929.45	2*h*			
2803.72	2				2929.71	4*h*			
2804.90	1				2930.16	2*h*			
2813.88	7				2934.4	2*hv*			
		2816.33	2*h*		2935.70	1			
2816.80	0						2936.72	4*h*	*Mg*?
2818.64	0				2941.38	1*h*			
2819.23	1				2947.25	2			
2820.57	1*h*				2949.18	1			
2821.31	0*h*				2949.96	2			
2825.19	1				2951.24	0			
2825.57	2				2961.77	1			
2827.32	1				2970.62	1			
2834.14	3				2972.20	1			
2840.10	1*h*				2972.88	5			
2841.36	1*h*				2974.26	1			
2843.13	1*hr*				2976.72	0			
2845.20	0*H*				2979.25	1			
2847.48	2				2982.85	2			
2849.18	1				2983.74	1			
2850.47	1				2988.61	7			
2859.36	2				2989.30	6			
2860.56	1				2990.24	1			
2866.57	7				2993.69	1			
2869.95	7				2994.97	1*h*	2994.96	3*s*	
2870.98	2				2996.47	1			
2873.74	0*H*				2997.23	4	2997.1	1*h*	
2877.92	1				2998.77	1			
2880.87	2				2999.51	1	2999.62	2	
2881.80	7				3000.82	1			
2882.48	1				3001.40	1			
2883.38	2				3002.02	2			
2884.15	1			*A*?	3002.53	0			
2885.54	1*h*				3004.16	2			
2887.25	1*h*				3005.19	0			
2891.96	2				3006.06	0			
2892.41	1*h*				3006.82	1	3006.86	4*s*	
2894.93	1*h*				3008.28	1			
2899.78	9				3009.27	1	3009.20	3*s*	
2905.59	0*h*				3011.84	1			
2907.90	2				3012.40	1			

TABLE I—*Continued*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
3018.05	0				3118.07	0			
3020.08	1*hd*				3119.66	8			
3021.74	1				3123.22	1			
3023.97	0				3129.35	1			
3024.70	4				3133.34	3			
3025.69	1				3134.57	1			
3028.66	6				3136.27	0			
3030.22	1*h*				3137.50	1			
3035.81	2				3138.04	0			
3041.47	3				3139.28	0			
3041.81	2				3142.77	1			
3043.34	3				3143.81	1			
3048.17	1				3144.34	0			
3049.42	2				3144.97	1			
3052.03	0				3145.76	2			
3054.95	0				3149.55	2			
3057.95	1				3151.21	1			
3059.76	0				3154.78	2*h*			
3060.27	1				3156.82	5			
3063.15	1				3158.87	9*hr*	3158.88	10*Rr*	
3063.49	2				3163.11	1			
		3066.28	3*h*		3165.78	4*h*			
3068.82	1				3167.87	1			
3071.39	1						3170.1	1*h*	
		3072.94	1*s*		3171.75	1			
3073.18	1				3172.75	2			
		3075.27	2*s*		3175.12	2			
3076.68	2				3179.34	10*hr*	3179.31	10*Rr*	
		3078.55	3*s*		3181.27	6*r*	3181.28	8*Rr*	
3079.10	2*h*				3188.72	1			
		3082.09	8*s*		3191.29	2			
3082.86	1				3194.76	6			
3083.69	1				3200.83	2			
3086.23	1				3201.22	1			
		3087.99	4*s*		3203.21	1*h*			
3089.00	2				3205.08	1			
3090.91	1*d*?				3206.10	5			
		3092.72	8*s*		3207.21	6			
3093.47	1			*A*?	3208.71	1			
3096.29	1				3211.37	1			
3096.89	1				3212.50	1			
3099.22	1				3213.89	0			
3099.80	2				3214.83	0			
3102.96	1				3216.33	2*h*			
		3103.80	2*s*				3216.92	1	
3104.43	0						3218.32	1	
3106.01	0				3220.75	6			
3106.95	1						3222.87	2*s*	
3107.49	1						3224.29	1*s*	
3115.10	0				3224.34	2			
3115.49	1				3228.02	2			
3116.43	0				3228.54	1			

TABLE I—*Continued*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
3233.02	4				3324.38	1			
3233.65	2				3325.35	0			
		3234.55	6*s*		3326.43	0			
3235.04	1*h*				3327.47	0			
		3236.57	5*s*				3329.46	3*s*	
		3239.02	3*s*				3332.12	2*h*	
3240.15	1				3334.30	0			
3240.97	0						3335.16	2*s*	
3241.75	1				3336.14	0			*A*?
3244.04	1				3337.27	0			
3248.28	1				3340.42	2			
		3248.58	3*s*		3340.65	3			
3250.26	1				3342.27	2			
3251.85	1				3343.17	2			
3254.38	1						3349.06	4*s*	
3255.86	3						3349.43	6*s*	
3259.21	0*h*				3354.14	1			
3260.93	1*h*				3354.71	1			
		3261.56	4*s*		3355.36	1			
3264.08	4				3356.70	0*h*			
3265.31	1				3358.33	1*h*			
3270.48	2				3359.34	1*h*			
3270.83	4				3360.08	2			
3271.68	1				3360.76	1*h*			
3273.17	1						3361.23	6*s*	
3273.96	5				3363.64	1			
		3278.60	2*s*		3367.81	5*r*			
3281.48	3				3372.68	8	3372.79	6*s*	
3283.32	1*d*?				3382.0	0*hd*			
3285.32	1				3386.39	2			
3285.81	1			*A*?			3387.85	3*s*	
3289.85	0				3389.89	1			
3290.42	3				3390.50	1			
3292.51	1				3391.33	1			
3294.53	1*h*				3392.81	1			
3296.50	0				3393.40	1			
3297.21	0				3393.86	0			
3298.56	0*h*				3398.13	0*d*			
3299.81	0*h*				3399.09	0			
3301.52	3				3401.17	2			
3302.90	6				3406.61	4			
3305.70	0*h*				3408.39	1*v*			
3306.95	0				3409.64	0			
3307.92	0				3410.10	0			
3308.78	2*r*				3411.65	0			
3311.46	4				3414.60	2			
3317.53	1				3415.38	1			
3319.62	1				3417.72	3			
3320.39	1*h*				3420.81	1			
3322.30	1				3422.86	1			
		3322.95	6*s*		3424.92	1			
3323.92	2				3428.51	2			

TABLE I—*Continued*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
3437.05	1				3547.70	3			
3441.42	0*h*				3549.45	2			
3442.83	1				3554.85	2			
		3444.39	3*s*		3559.54	0*h*			*A*?
3445.86	0				3560.61	2			
3447.23	0				3563.85	0			
3448.31	1				3564.31	1			
3449.51	1				3565.14	0			*A*?
3450.18	1				3566.61	2*h*			
3451.03	1*d*				3569.56	0			
		3456.43	2*s*		3570.55	0			
3457.13	1				3571.30	0			
3462.62	1				3572.05	1			
3466.99	0				3573.06	0*r*			
3467.76	1				3579.47	3			
3473.09	0*h*				3581.31	4			
		3474.81	2*hr*		3582.18	1			
3476.11	1				3582.81	1			
3476.74	0			*A*?	3583.62	1			
3477.72	1				3585.87	0			
3478.63	1				3587.13	0	3587.01	4*h*	
3483.18	1*h*				3588.52	5			+*A*?
3486.40	1*hr*				3590.45	1			
		3487.72	2*hr*		3592.10	0			
3490.37	0				3593.25	0			
3491.47	1*h*			*A*?			3594.11	1*s*	
3495.23	0*h*				3594.64	0			
3500.26	4				3596.04	0			
3502.02	2						3601.81	2*h*	
3503.18	0				3602.06	3*v*			
		3504.85	5*s*		3605.20	4*v*			
3506.29	1*h*				3609.30	1*h*			
		3510.82	4*s*		3611.51	1*h*			
3511.08	0				3612.77	2			
3512.04	0				3613.73	1			
3513.29	0				3617.31	1*hr*			
3514.30	0				3622.67	1			
3515.43	0				3623.94	1*h*	3624.01	5*hv*	
3517.10	0				3624.94	1			
3518.69	1				3627.31	1			
3519.73	0				3628.69	0			
3522.71	5				3629.45	0			
3529.96	2				3630.68	1	3630.66	6*hv*	
3531.09	2				3631.51	4			
3533.02	1				3632.68	0			*A*?
		3535.45	2*s*		3635.95	1			
3537.75	7				3637.87	0*h*			
3539.92	1				3639.78	1*h*			*A*?
3540.36	1				3644.38	2	3644.32	8	
3541.40	2*h*				3645.08	1*h*			
3542.78	2*v*				3646.05	1			
3545.47	3				3647.88	2			

TABLE I—*Continued*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
3650.62	2*h*	.....	.....	.....	3762.63	2	.....	.....	.....
3652.56	2	.....	.....	.....	3772.06	1	.....	.....	.....
.....	.....	3653.46	2	.....	3773.14	0	.....	.....	.....
3654.93	0*h*	.....	.....	.....	3774.18	0	.....	.....	.....
3656.29	3	.....	.....	.....	3775.86	1	.....	.....	.....
3657.18	3	.....	.....	.....	3777.01	0	.....	.....	.....
3660.11	0	.....	.....	.....	3778.82	0*h*	.....	.....	.....
3661.44	0	.....	.....	.....	3780.86	6	.....	.....	*A*?+
3662.93	2	.....	.....	.....	3783.24	3	.....	.....	.....
3663.63	0	.....	.....	.....	3785.76	2*h*	.....	.....	.....
3667.00	0	.....	.....	.....	3788.81	0	.....	.....	.....
3669.15	0	.....	.....	.....	3789.64	2	.....	.....	.....
3670.20	1	.....	.....	.....	3790.25	1	.....	.....	.....
3679.71	1*v*	.....	.....	.....	3791.37	1	.....	.....	.....
3681.08	1	.....	.....	.....	3792.07	1	.....	.....	.....
3682.03	1	.....	.....	.....	3793.34	3	.....	.....	.....
3683.44	0*h*	.....	.....	.....	3794.39	2	.....	.....	.....
3684.59	2	.....	.....	.....	3796.12	2	.....	.....	.....
.....	.....	3685.17	3*s*	.....	3797.79	1	.....	.....	.....
3686.67	2	.....	.....	.....	3798.84	0	.....	.....	.....
3690.84	4*v*	.....	.....	.....	3799.93	4	.....	.....	.....
3693.07	0	.....	.....	.....	3800.93	1	.....	.....	.....
3694.26	0	.....	.....	.....	3801.89	1*r*	.....	.....	.....
3695.30	0	.....	.....	.....	3805.17	1*v*	.....	.....	.....
.....	.....	3696.28	2*hr*	.....	3806.50	2	.....	.....	.....
3699.78	0	.....	.....	.....	3808.84	0	.....	.....	.....
3701.15	1	.....	.....	.....	3810.11	1	.....	.....	.....
3702.44	1	.....	.....	.....	3810.99	0	.....	.....	.....
3704.04	1	.....	.....	.....	3813.01	0	.....	.....	.....
3706.00	10*hr*	3706.04	10*hr*	.....	3814.66	1	.....	.....	.....
3710.76	2*v*	.....	.....	.....	3820.28	0	.....	.....	.....
3712.37	2	.....	.....	.....	3822.01	0	.....	.....	.....
3715.87	2	3716.04	2*hr*	.....	3823.76	4	.....	.....	.....
3717.03	1	.....	.....	.....	3824.43	0	.....	.....	.....
3720.40	2	.....	.....	*A*?	3826.12	0*h*	.....	.....	.....
3723.87	2*h*	.....	.....	.....	.....	.....	3826.36	4*s*	.....
3725.75	1	.....	.....	.....	3827.61	0*h*	.....	.....	.....
3726.60	0	.....	.....	.....	3829.06	1*H*	.....	.....	.....
3728.46	3	.....	.....	.....	3832.26	1	.....	.....	.....
3732.00	4	.....	.....	.....	3833.27	1	.....	.....	.....
3733.92	2	.....	.....	.....	3836.49	4*h*	.....	.....	.....
3734.91	1	.....	.....	.....	3838.31	2	.....	.....	.....
3736.93	11*hr*	3736.94	10*hr*	.....	3842.55	4	.....	.....	.....
.....	.....	3747.00	4*h*	.....	3845.32	1	.....	.....	.....
3748.89	0	.....	.....	.....	3846.15	1	.....	.....	.....
3750.15	2	.....	.....	.....	3850.93	1*h*	.....	.....	.....
3751.14	1*r*	.....	.....	.....	3852.49	2*h*	.....	.....	.....
3756.57	1	.....	.....	.....	3856.03	1*s*	3856.00	2*h*	.....
3757.27	0	.....	.....	.....	3858.81	3*h*	.....	.....	.....
3758.71	3	.....	.....	.....	3859.67	5*h*	.....	.....	.....
.....	.....	3759.27	3*s*	.....	3861.09	2*h*	.....	.....	.....
3761.62	6	.....	.....	.....	3862.59	0	.....	.....	.....

TABLE I—*Continued*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
3868.42	3	.....	.....	.....	3990.69	2	.....	.....	.....
3869.46	1	.....	.....	.....	3991.46	0	.....	.....	.....
3870.42	4	.....	.....	.....	3992.46	0	.....	.....	.....
3885.29	4*h*	.....	.....	.....	3993.10	0	.....	.....	.....
3890.82	2	.....	.....	.....	3994.26	1	.....	.....	.....
3892.03	0	.....	.....	.....	3999.13	1	.....	.....	.....
3893.43	0	.....	.....	.....	4003.35	5	.....	.....	.....
3894.66	1	.....	.....	.....	4006.29	1	.....	.....	.....
3896.03	0	.....	.....	.....	4010.26	2*h*	.....	.....	.....
3897.22	6	.....	.....	.....	4013.88	1	.....	.....	*A*?
3898.53	0	.....	.....	.....	4016.15	1*h*	.....	.....	.....
3899.65	0	.....	.....	.....	4018.89	4	.....	.....	.....
3900.92	0	.....	.....	.....	4024.68	4	.....	.....	.....
3902.22	1*d*?	.....	.....	.....	4030.3	0*H*	.....	.....	.....
3902.98	0	.....	.....	.....	4037.18	1	.....	.....	.....
3903.95	0	.....	.....	.....	4038.53	3	.....	.....	.....
3905.03	0	.....	.....	.....	4040.3	0*H*	.....	.....	.....
.....	.....	3905.54	4*s*	.....	4044.44	0	.....	.....	*A*?
.....	.....	3909.83	1	.....	4047.34	4	.....	.....	.....
3910.31	1	.....	.....	.....	4051.11	0*h*	.....	.....	.....
3912.74	1*v*	.....	.....	.....	4054.64	0*h*	.....	.....	.....
3914.13	2*h*	.....	.....	.....	4057.10	1	.....	.....	.....
.....	.....	3915.24	1	.....	.....	.....	4057.83	3*s*	.....
3918.59	3	.....	.....	.....	4059.84	4	.....	.....	.....
.....	.....	3923.20	10*R*	*g*?	4064.67	0*h*	.....	.....	.....
3923.75	2	.....	.....	.....	4066.80	0*h*	.....	.....	.....
3925.71	1*h*	.....	.....	*A*?	4069.01	4	.....	.....	.....
3929.33	1	.....	.....	.....	4071.30	2	.....	.....	.....
3931.12	1	.....	.....	.....	4081.74	5	.....	.....	.....
3932.07	1	.....	.....	.....	4088.82	4	.....	.....	.....
3933.65	13	3933.65	10*R*	.....	4092.80	2*h*	.....	.....	.....
3942.84	1	.....	.....	.....	.....	.....	4095.09	2*hr*	.....
3944.25	1	.....	.....	*A*?	.....	.....	4098.73	2*hr*	.....
3945.81	1*d*	.....	.....	.....	4099.01	2	.....	.....	.....
3946.42	1	.....	.....	.....	4109.98	1*r*	.....	.....	.....
3947.34	1	.....	.....	.....	4114.19	4*r*	.....	.....	.....
.....	.....	3948.95	3*s*	.....	4116.04	3	.....	.....	.....
3949.57	4	.....	.....	.....	.....	.....	4123.24	1*H*	.....
3954.64	2	.....	.....	.....	.....	.....	4127.81	1*H*	.....
3957.01	1	3957.08	4*s*	.....	4128.07	0	.....	.....	.....
3957.87	1	3957.81	*R*	*g*?	4129.12	1*d*?	.....	.....	.....
3958.92	1	.....	.....	.....	4130.43	1*d*	.....	.....	.....
3960.46	2	.....	.....	.....	.....	.....	4130.83	1*H*	.....
3962.28	5	.....	.....	.....	4132.51	1	.....	.....	.....
3968.46	12	3968.49	10*R*	.....	4136.33	6*r*	.....	.....	.....
.....	.....	3973.76	2*s*	.....	4141.96	1	.....	.....	.....
3977.82	4	.....	.....	.....	4144.16	3	.....	.....	.....
.....	.....	3979.06	*R*	*g*?	4147.70	1	.....	.....	.....
3980.31	4	.....	.....	.....	4148.37	3	.....	.....	.....
3984.71	6	.....	.....	.....	4151.33	0	.....	.....	.....
3988.72	1*v*	.....	.....	.....	4153.63	7	.....	.....	.....
3989.66	0	.....	.....	.....	4156.98	0*H*	.....	.....	.....

TABLE I—*Continued*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
4159.12	0*H*	.....	.....	.....	4321.54	1	.....	.....	.....
4161.82	1*h*	.....	.....	.....	4325.68	0*h*	.....	.....	.....
4164.37	6*h*	.....	.....	.....	4329.22	7*hv*	.....	.....	.....
4165.48	2*h*	.....	.....	.....	.....	.....	4330.15	2	.....
4166.87	1*h*	.....	.....	.....	.....	.....	4333.77	1	.....
4169.60	2*hr*	.....	.....	.....	4341.09	1	.....	.....	.....
4175.68	6	.....	.....	.....	4343.39	1	.....	.....	.....
4178.73	0	.....	.....	.....	4344.73	2*h*	.....	.....	.....
4179.74	1	.....	.....	.....	4348.06	3*s*	.....	.....	*A*?
4180.69	1	.....	.....	.....	4352.28	0	.....	.....	*A*?
4184.29	8*r*	.....	.....	.....	.....	.....	4355.30	3*hr*	.....
4194.22	3	.....	.....	.....	4358.42	6	.....	.....	.....
4196.60	1*h*	.....	.....	.....	4362.88	1	.....	.....	.....
4199.08	2	.....	.....	.....	4364.12	1	.....	.....	.....
4201.77	1	.....	.....	.....	4366.61	2	.....	.....	.....
4204.63	2	.....	.....	.....	4369.48	0	.....	.....	.....
4207.26	10	.....	.....	.....	4370.73	1	.....	.....	.....
4211.65	2	.....	.....	.....	4371.28	1	.....	.....	*A*?
4213.15	5	.....	.....	.....	4374.61	1*hv*	.....	.....	.....
4226.75	10	4226.71	8*R*	.....	4381.19	3	.....	.....	.....
4228.23	3*h*	.....	.....	.....	4382.93	2	.....	.....	.....
4233.73	8*h*	.....	.....	.....	4393.16	2	.....	.....	.....
.....	.....	4238.43	1*H*	.....	4394.00	1	.....	.....	.....
.....	.....	4240.36	2*h*	.....	4395.99	0	.....	.....	.....
4240.74	8*h*	.....	.....	.....	4399.64	10	.....	.....	.....
4246.10	1	.....	.....	.....	4406.33	8	.....	.....	.....
4248.95	0	.....	.....	.....	4413.74	3	.....	.....	.....
4255.53	0	.....	.....	.....	4419.13	1	.....	.....	.....
4256.69	3	.....	.....	.....	4420.73	0	.....	.....	.....
4258.33	1	.....	.....	.....	4425.43	4*hv*	4425.45	8*h*	.....
.....	.....	4271.60	1*h*	.....	4425.99	2	.....	.....	*A*?
4271.87	7*h*	.....	.....	.....	4431.33	7	.....	.....	.....
4273.90	5	.....	.....	.....	4434.93	4*hv*	4434.95	6*h*	.....
.....	.....	4277.24	1	.....	4435.71	2	4435.67	6*s*	.....
.....	.....	4277.86	2	.....	4438.64	0*h*	.....	.....	.....
4278.28	6*h*	.....	.....	.....	.....	.....	4442.79	1	.....
4279.76	7*h*	.....	.....	.....	.....	.....	4443.92	1	.....
4283.00	5*h*	4282.97	8	.....	4450.75	0	.....	.....	.....
4284.40	7*h*	.....	.....	.....	4454.75	5*hv*	4454.75	7*h*	.....
4289.39	6*h*	4289.37	8	.....	4455.83	3*hv*	4455.89	7*s*	.....
4290.12	6*h*	.....	.....	.....	4456.5	1?	4456.62	3*s*	.....
4295.61	3	.....	.....	.....	.....	.....	4466.46	1*h*	.....
4297.26	1	.....	.....	.....	.....	.....	4467.76	1*h*	.....
4299.00	4*v*	4298.97	8	.....	4479.44	3	.....	.....	.....
4300.97	5	.....	.....	.....	.....	.....	4481.17	2*h*	*Mg*?
4301.54	5	.....	.....	.....	4484.41	3	.....	.....	.....
4302.76	10	4302.52	9	.....	4484.96	5	.....	.....	.....
4307.73	5	4307.70	5	.....	4487.44	0*h*	.....	.....	.....
.....	.....	4310.43	1	.....	4488.68	0*h*	.....	.....	.....
.....	.....	4313.99	2	.....	4493.92	1*h*	.....	.....	.....
4318.63	5	4318.64	8*hv*	.....	4496.03	1*h*	.....	.....	.....
4319.54	3	.....	.....	.....	4498.07	1*h*	.....	.....	.....

TABLE I—*Continued*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
4499.90	10*h*				4776.76	0			
4505.20	3*h*				4779.35	0			
4508.81	4				4794.50	2*s*			*Cl?*
4512.88	4				4806.01	1*s*			*A?*
4516.60	8				4809.99	1*s*			*Cl?*
4522.36	1*h*				4817.09	0*h*			
4526.8	1*H*	4527.00	4*hr*		4818.33	0			
4535.57	1*h*				4819.46	0			*Cl?*
4545.06	0			*A?*			4847.0	1*h*	
4549.19	0				4848.13	1*h*			
4552.64	3				4852.16	2*h*			
4553.27	4*h*				4859.29	3*h*			
4554.60	1				4874.03	0*h*			
4564.43	4*h*				4878.15	1*h*	4878.18	4*s*	
4566.35	2*h*				4879.83	0			*A?*
4567.79	2				4889.96	2*r*			
4572.11	8*h*				4896.88	0			
4576.57	4*h*				4899.21	0			
		4578.60	4*s*		4919.43	2*r*			
4581.65	2*hr*	4581.44	5*s*		4922.73	1			
4586.03	3*h*	4585.91	6*hr*		4950.66	1*hv*			
4586.82	1				4986.14	0			
4588.9	4*H*				4989.32	0			
4597.4	4*H*				5009.02	2			
4600.40	3*h*				5014.23	1			
4608.54	2						5041.74	1*h*	
4609.50	1*s*			*A?*	5046.95	1			
4621.70	5*hv*				5050.04	2			
4629.68	1				5063.73	0			
4631.39	4*h*				5071.89	0			
4632.67	4*h*				5137.78	1			
4634.22	1				5188.72	0	5188.81	2*s*	
4642.88	4*h*				5201.41	0			
4643.66	4*h*				5231.83	2			
4650.4	3*H*				5247.42	3			
4655.5	0*H*						5261.69	3*s*	
4663.45	1*h*				5262.18	1	5262.20	3*s*	
4665.15	1*h*				5264.08	0	5264.23	3*s*	
4667.34	1*h*				5265.57	1*r*	5265.55	5*s*	
4671.72	0*H*				5270.30	5	5270.29	5*s*	
4673.71	0*h*				5271.97	5			
4679.74	1*h*				5294.48	0			
4682.92	0				5301.34	1			
4689.31	0				5321.30	3			
4701.40	1*h*				5328.01	2			
4708.81	5				5333.78	0			
4716.26	4				5349.39	1	5349.43	5*s*	
4720.25	2				5361.43	0			
4727.70	0				5495.95	0			
4735.94	1						5512.90	2*hv*	
4736.78	2*hr*				5570.49	1			
4760.24	0				5579.13	1			

TABLE I—*Continued*

JAA		E AND V			JAA		E AND V		
λ	Int.	λ	Int.	R	λ	Int.	λ	Int.	R
5582.07	1	5581.95	4s	.....	6173.12	0	.....	.....	.....
5588.72	3	5588.73	6h	.....	6200.72	0	.....	.....	.....
5590.09	1h	5590.10	4s	.....	6214.10	1	.....	.....	.....
5594.43	2	5594.41	6s	.....	6294.90	0	.....	.....	.....
5598.41	2	5598.46	5s	.....	6370.15	0	.....	.....	.....
5601.30	1	5601.26	5s	.....	6387.59	0	.....	.....	.....
5602.95	1	5602.79	5s	.....	6424.52	1	.....	.....	.....
5752.64	1	.....	.....	.....	6432.07	0h	.....	.....	.....
5846.03	0	.....	.....	.....	6439.07	4	6439.2	10	.....
5857.49	1hr	5857.5	8	.....	6449.84	1	6449.78	8	.....
6069.98	2	.....	.....	.....	6462.58	3	6462.6	10	.....
6102.85	0	6102.78	8	.....	6471.68	1	6471.7	8	.....
6122.23	1	6122.3	10	.....	6476.74	0	.....	.....	.....
6162.22	1r	6162.3	10	.....	6485.38	1	.....	.....	.....
.....	.....	6166.6	5	.....	6493.88	2	6493.7	10	.....
.....	.....	6169.2	5	.....	6499.70	1	6499.7	8	.....
.....	.....	6169.7	5	.....					

20 lines which agreed closely enough in wave-length, intensity, and character to be regarded as identical; and all these lines were weak. In preparing for this work, however, a series of spectrograms was made, using various metallic terminals in the same vacuum chamber, but at a pressure ranging from several centimeters down to about 0.1 mm, for the purpose of studying the air spectrum as excited by these powerful sparks. From a study of these spectrograms, a list of about 450 lines was prepared, covering the region from λ 2900 to λ 5000. Most of the oxygen and nitrogen lines given by Lunt, Neovius, Exner and Haschek, and Fowler were identified, but considerable differences in the intensities were found. This list has been very useful in the identification of air lines in the vacuum spark spectrum, and it has brought out very clearly the fact that at low pressures the air spectrum is different with different electrodes. To give but a few illustrations: The great nitrogen pair λ 4097 and λ 4103 is present with calcium, absent with cadmium, and extremely weak, if present at all, with carbon. The enhanced oxygen lines λ 3754 and λ 3760 are fairly conspicuous with aluminum, weak with calcium and carbon, and extremely weak, if present, with cadmium. With both calcium and carbon the intensities of this pair are reversed, λ 3754 being very much stronger than λ 3760.

The enhanced oxygen line λ 3961 is absent from all the spectra examined except *Mg*, and the enhanced nitrogen line λ 4379 is absent in all cases.

TABLE II

JAA		E AND V		CLASSIFICATION
λ	Int.	λ	Int.	
3158.87	9*hr*	3158.88	10*Rr*	1π–2δ *Ca*+
3179.34	10*hr*	3179.31	10*Rr*	1π–2δ *Ca*+
3181.27	6*r*	3181.28	8*Rr*	1π–2δ *Ca*+
3706.00	10*hr*	3706.04	10*hr*	1π–2σ *Ca*+
3736.93	11*hr*	3736.94	10*hr*	1π–2σ *Ca*+
3933.65	13	3933.65	10*R*	1σ–1π *Ca*+
3968.46	12	3968.49	10*R*	1σ–1π *Ca*+
4226.75	10	4226.71	8*R*	1S–1P *Ca*
4283.00	5*h*	4282.97	8	
4289.39	6*h*	4289.37	8	
4299.00	4*v*	4298.97	8	
4302.76	10	4302.52	9	
4307.73	5	4307.70	5	
4318.63	5	4318.64	8*hv*	
4425.43	4*hv*	4425.45	8*h*	1p–2d *Ca*
4434.93	4*hv*	4434.95	6*h*	1p–2d *Ca*
4435.71	2	4435.67	6*s*	1p–2d *Ca*
4454.75	5*hv*	4454.75	7*h*	1p–2d *Ca*
4455.83	3*hv*	4455.89	7*s*	1p–2d *Ca*
4456.5	1?	4456.62	3*s*	1p–2d *Ca*
4581.65	2*hr*	4581.44	5*s*	1d–3f *Ca*
4586.03	3*h*	4585.91	6*hr*	1d–3f *Ca*
4878.15	1*h*	4878.18	4*s*	1D–3F *Ca*
5188.72	0	5188.81	2*s*	1P–3D *Ca*
5262.18	1	5262.20	3*s*	
5264.08	0	5264.23	3*s*	
5265.57	1*r*	5265.55	5*s*	
5270.30	5	5270.29	5*s*	
5349.39	1	5349.43	5*s*	
5582.07	1	5581.95	4*s*	
5588.72	3	5588.73	6*h*	
5590.09	1*h*	5590.10	4*s*	
5594.43	2	5594.41	6*s*	
5598.41	2	5598.46	5*s*	
5601.30	1	5601.26	5*s*	
5602.95	1	5602.79	5*s*	
5857.49	1*hr*	5857.5	8	
6102.85	0	6102.78	8	1p–1s *Ca*
6122.23	1	6122.3	10	1p–1s *Ca*
6162.22	1*r*	6162.3	10	1p–1s *Ca*
6439.07	4	6439.2	10	
6449.84	1	6449.78	8	
6462.58	3	6462.6	10	
6471.68	1	6471.7	8	
6493.88	2	6493.7	10	
6499.70	1	6499.7	8	

The atmospheric lines have been excluded from Table I as far as the available data permit. No doubt a few weak air lines have been missed, and perhaps some lines really belonging to calcium have been removed. A few lines which agree closely in wave-length with lines of argon are indicated by *A* ? in the column headed R. The table also gives three lines near λ 4800 which are probably due to chlorine. The appearance of argon lines when calcium electrodes are used is, of course, to be expected, since the noble gases are not absorbed to any appreciable extent by the hot metal. If the lines marked *A* ? are really due to argon, the selection may be of some significance; there are many strong lines in the argon spectrum which do not appear here, while some of the lines which do appear are argon lines of only moderate intensity.

There remain to be discussed the lines due to the electrode points *FF*. The investigation so far has shown that some known arc and spark lines of the element always appear in the vacuum spark. It should accordingly be possible to detect impurities by their well-known strong lines. Magnesium is present, as shown by λ 2852, with intensity 2, and the *Mg*+ lines λ 2795 and λ 2802, each with intensity 5, and λ 2790 and λ 2798 with intensity 3. A few other faint *Mg* lines were found, among them λ 4481. Strontium is represented by λ 4607, the enhanced pair λ 4077 and λ 4215, and a few other lines, mostly very faint. No lines of barium were found.

There is a question whether some lines in the table may belong to the vacuum spark spectrum of elements other than calcium. This can be answered in the negative as far as magnesium is concerned, since its vacuum spark spectrum has been photographed in this investigation. Some strontium lines of this class may be present, but in all probability they would be rather weak. No such lines of other elements should occur, since their strong arc and spark lines are absent. It may reasonably be concluded, therefore, that the new lines in Table I are due chiefly to calcium.

The author wishes to thank Mr. Sinclair Smith for his assistance in carrying out this investigation.

Mount Wilson Observatory
October 15, 1923

THE ABSOLUTE MAGNITUDES OF LONG-PERIOD VARIABLE STARS[1]

By PAUL W. MERRILL and GUSTAF STRÖMBERG

ABSTRACT

Absolute magnitudes of long-period variables.—The mean value of the absolute magnitude at maximum computed from van Maanen's *trigonometric parallaxes* is +0.4, corresponding to spectral type M6.6e. A study of *proper motions* yields absolute magnitudes of −0.5 for stars of classes M1e–M5e, +0.5 for classes M6e–M8e, and +0.4 for class Se. Modified formulae for the computation of absolute magnitude from proper motions are discussed in some detail. The *character of the spectrum* also indicates that stars of classes Me and Se are giants. One component of the *variable double star X Ophiuchi* is of class M6e, and has an absolute magnitude of +0.3 at maximum. The adopted mean value for all stars of class Me is +0.1. Reasonable assumptions as to the surface brightness and the total radiation per unit area lead to computed diameters of about 130 times that of the sun. The angular diameters should thus be somewhat greater than the parallaxes, and one or two of the nearest stars may be measurable with the stellar interferometer.

Average speed and solar motion.—The apparent solar speed is found to increase with the average residual stellar velocity. This is in harmony with previous results for other classes of stars.

The problem of the intrinsic luminosities of long-period variable stars can profitably be discussed at present because several investigations bearing on it have recently been completed. Several means of attack are available; namely, (1) trigonometric parallaxes, (2) comparison of proper motion with linear motion, (3) character of the spectrum, and (4) special cases of double or companion stars.

Throughout this paper the magnitudes will refer to the maxima of the mean light-curves.

TRIGONOMETRIC PARALLAXES

The parallaxes of very few long-period variables have been determined by the trigonometric method. *o* Ceti has been observed by several investigators, but it is not safe to use their results because recent spectroscopic observations[2] have shown that the telescopic image is not that of a point, and that the position of the effective center of the image varies with the light-phase. The only other

[1] *Contributions from the Mount Wilson Observatory*, No. 267.

[2] *Popular Astronomy*, **31**, 237, 1923.

trigonometric parallaxes available are those of van Maanen for the stars listed in Table I. These data are taken from *Contribution* No. 237, with the exception of those for U Herculis, which, through the kindness of Mr. van Maanen, were supplied in advance of publication.

TABLE I

TRIGONOMETRIC PARALLAXES OF LONG-PERIOD VARIABLES

Designation	Star	Spec.	π abs.	p.e.	m
001755	T Cassiop.	M8e	+0".026	±0".003	8.3
023133	R Trianguli	M5e	+0.004	0.006	6.2
081112	R Cancri	M7e	−0.002	0.005	7.0
123307	R Virginis	M6e	+0.009	0.004	7.1
134440	R Can. Ven.	M6e	−0.011	0.003	7.6
162119	U Herculis	M8e	+0.006	0.006	7.7
183308	X Ophiuchi	M6e	−0.001	0.005	6.8

To find the mean absolute magnitude from the trigonometric parallaxes the following formula has been used:

$$\bar{M} = 5 + 5[\log \Sigma\pi - \log \Sigma\ 10^{-0.2m}] \qquad (1)$$

where π denotes the trigonometric parallax and m the apparent magnitude. This formula is based on the assumption that the absolute magnitudes of all the stars in the group are the same, and that the mean of the parallaxes corresponding to this constant absolute magnitude is equal to the mean of the trigonometric parallaxes; or, what amounts to the same thing, that the sum of the errors of the trigonometric parallaxes is zero. The alternative condition, that the sum of the squares of the errors of the trigonometric parallaxes be a minimum, leads to the equation

$$\bar{M} = 5 + 5[\log \Sigma\pi\ 10^{-0.2m} - \log \Sigma\ 10^{-0.4m}]\,. \qquad (2)$$

This equation is not recommended for use in the present case as it would in effect give too much weight to the stars that are apparently the brightest.

The value found from equation (1), giving all the stars equal weight, is

$$\bar{M} = +0.4.$$

The mean spectral class is M6.6e.

COMPARISON OF PROPER MOTION WITH LINEAR MOTION

A study of the radial velocities of long-period variables[1] has shown that if the stars be grouped according to their peculiar radial velocities, the group having the higher internal velocity will give the greater apparent solar motion. Further, the average internal radial velocity (and hence the solar motion) varies with spectral type. The position of the solar apex does not deviate greatly from the usual position.

For the purpose of the present investigation we have determined the speed of the sun with respect to groups of stars selected by their spectral types, assuming the apex to lie in the position $\alpha = 270°$, $\delta = +30°$. The results are shown in Table II, in which V_0 is the sun's velocity and θ the mean value of the residual velocities, the signs being disregarded. The increase of V_0 with θ and their common dependence on spectral class are evident.

TABLE II

SOLAR MOTION AND SPECTRAL CLASS

Spectrum	No. Stars	Mean Spectrum	V_0	θ
			km/sec.	km/sec.
M2, 3, 4e	35	M3.3e	64.2±8.6	38.3
M3, 4, 5e	49	M4.1e	71.8±7.0	39.5
M5, 6e	49	M5.6e	62.3±6.0	31.7
M6, 7e	50	M6.4e	38.7±5.2	26.5
M7, 8e	34	M7.4e	25.6±5.6	23.1
Se	14	Se	21.9±8.6	19.1

The increase of apparent solar motion with the random stellar motion is in harmony with previous results for other classes of stars,[2] and is an example of an asymmetry in the distribution of stellar velocities, which appears to be of very great generality.

The values of V_0 and θ from Table II can be combined with proper motions to determine the mean parallaxes of the various spectral groups. Through the work of R. E. Wilson,[3] proper

[1] *Mt. Wilson Contr.*, No. 264; *Astrophysical Journal*, **58**, 215, 1923.

[2] *Mt. Wilson Contr.*, No. 245; *Astrophysical Journal*, **56**, 265, 1922; *Astronomical Journal*, **35**, 26, 1923; *Proceedings of the National Academy of Sciences*, **9**, 312, 1923.

[3] *Astronomical Journal*, **34**, 183, 1923.

motions are available for seventy stars of class Me for which the spectral subdivision has recently been determined at Mount Wilson, and for ten stars of class Se.

The method of finding the absolute magnitudes is practically the same as that used by one of us in *Contribution* No. 199. The formulae have been modified, however, to avoid the use of a preliminary value of the absolute magnitude, and slight changes in the notation have been introduced. The revised formulae for parallactic motions are collected below:

α, δ, right ascension and declination of the star

$\mu_1 = 15\,\mu_\alpha^s \cos\delta$; $\mu_2 = \mu_\delta$ components of star's proper motion in seconds of arc per year

π, star's parallax

x_0, y_0, z_0, linear components of standard solar motion in the equatorial system in astronomical units per year. The adopted solar motion is 20 km/sec. toward $\alpha = 270°$, $\delta = +30°$

$x_0 = \quad 0.000$ astron. units per year $= \quad 0.00$ km/sec.
$y_0 = -3.656$ astron. units per year $= -17.32$ km/sec.
$z_0 = +2.111$ astron. units per year $= +10.00$ km/sec.

Put

$$\left.\begin{aligned} A &= x_0 \sin\alpha - y_0 \cos\alpha \\ B &= x_0 \cos\alpha \sin\delta + y_0 \sin\alpha \sin\delta - z_0 \cos\delta \end{aligned}\right\} \qquad (3)$$

In the present case these auxiliary quantities become

$$\left.\begin{aligned} A &= +3.656 \cos\alpha \\ B &= -3.656 \sin\alpha \sin\delta - 2.111 \cos\delta \end{aligned}\right\} \qquad (3')$$

for which tables may be constructed. The equations of condition for finding the mean parallax of a group of stars now become

$$\left.\begin{aligned} A\pi &= \mu_1 \\ B\pi &= \mu_2 \end{aligned}\right\} \qquad (4)$$

which lead to the normal equation

$$\bar{\pi} = \frac{\Sigma(A\mu_1 + B\mu_2)}{\Sigma(A^2 + B^2)} . \qquad (5)$$

This formula is really identical with the ordinary mean parallax formula

$$\bar{\pi}=\frac{\Sigma v \sin \lambda}{V_0 \Sigma \sin^2 \lambda} \tag{6}$$

since

$$\left.\begin{aligned} v&=\frac{A\mu_1+B\mu_2}{\sqrt{A^2+B^2}}, \qquad \tau=\frac{A\mu_2-B\mu_1}{\sqrt{A^2+B^2}}, \\ \sin\lambda&=\frac{\sqrt{A^2+B^2}}{V_0} \end{aligned}\right\} \tag{7}$$

but the numerical computation by (5) is very much shorter.

In finding the mean absolute magnitude of a group of stars, some assumption must be made concerning the distribution of the absolute magnitudes of the individual stars of the group. If the range is thought to be small, a constant absolute magnitude may be assumed. In this case we have for each star

$$\pi=0.1\times 10^{0.2(M-m)},$$

where M and m represent the (constant) absolute and apparent magnitude, respectively. It is evident from this equation that in dealing with stars of different apparent magnitudes we should reduce all to the basis of a common apparent magnitude m_0 by multiplying the proper motions by

$$\rho=10^{0.2(m-m_0)}.$$

We have then

$$\left.\begin{aligned} A\pi_0&=\rho\mu_1 \\ B\pi_0&=\rho\mu_2 \\ M&=m_0+5+5\log\pi_0. \end{aligned}\right\} \tag{8}$$

If it is desired to use a solar velocity V_s differing from the standard velocity V_0 (20.0 km/sec.), it is necessary to apply to the absolute magnitude the correction

$$\Delta M=5\ (\log V_0-\log V_s). \tag{9}$$

The mean parallaxes of several groups of stars, computed from the parallactic motions, are given in the fifth column of Table III. These parallaxes correspond to apparent magnitude 5.0.

TABLE III

ABSOLUTE MAGNITUDES FROM PROPER MOTIONS

Spectrum	No.	Mean Spectrum	V_0	π_{par}	θ	π'_{pec}	π''_{pec}	M
			km/sec.		km/sec.			
M1, 2, 3, 4e.	20	M3.4e	64.2	+0".0044	38.3	+0".0088	+0".0061	−0.6
M3, 4, 5e...	31	M4.1e	71.8	+0.0023	39.5	+0.0073	+0.0088	−0.5
M5, 6e....	29	M5.6e	62.3	+0.0023	31.7	+0.0101	+0.0143	+0.4
M6, 7e....	29	M6.4e	38.7	+0.0071	26.5	+0.0128	+0.0148	+0.6
M7, 8e....	21	M7.1e	25.6	+0.0173	23.1	+0.0116	+0.0148	+0.6
Se.....	10	Se	21.9	+0.0124	19.1	+0.0026	+0.0112	+0.4

Russell[1] has investigated the errors in the mean parallaxes of a group of stars as computed from the parallactic motion and from the peculiar motion. These errors are in the ratio

$$r_{par}/r_{pec} = 1.44\,\theta/V_0 .$$

This formula would indicate greater accuracy for the value derived from the parallactic motion for the first three groups in Table III, and for the values derived from the peculiar motions for the last two groups. Inspection of the residuals for individual stars in the respective groups indicates, however, that the peculiar motion gives the more reliable values in all cases, except for the S stars. This probably arises from errors in the accepted values of V_0 for the various groups. The value of θ, on the other hand, is only slightly affected by a change in V_0.

If the τ-components are used to compute the mean parallax, the formula is

$$\pi_0 = \frac{k\Sigma\rho\tau}{n\theta}, \qquad (10)$$

where $k = 4.738$ km/sec., and n is the number of stars in the group. A convenient formula for τ is found in (7).

[1] *Mt. Wilson Contr.*, No. 215; *Astrophysical Journal*, **54**, 140, 1921.

Corresponding formulae for the proper-motion components μ_1 and μ_2 are[1]

$$\left.\begin{aligned} k\Sigma\left|\rho\mu_1-\frac{V_s A\pi_0'}{20}\right| &= \pi_0 n\theta \\ k\Sigma\left|\rho\mu_2-\frac{V_s B\pi_0'}{20}\right| &= \pi_0 n\theta. \end{aligned}\right\} \qquad (11)$$

π_0' is an assumed value of the parallax; in the present case that corresponding to $M=0$. V_s and θ are here expressed in km/sec.

The proper motions of the Me stars are very small, and errors in the determinations are doubtless of considerable size compared to the real motions. To correct for the systematic effect of the accidental errors of measurement, we may assume the distribution of the peculiar proper-motion components to correspond to a normal error curve, and then subtract the extra dispersion caused by the errors of measurement. For example, instead of equation (10), we may write

$$\pi_0=\frac{k}{\theta}\sqrt{\left(\frac{\Sigma\rho\tau}{n}\right)^2-\eta^2}\ , \qquad (12)$$

where η is the average error in the reduced τ-component, a quantity which can easily be computed from the published probable errors in μ_1 and μ_2. A similar modification must be applied to equations (11).

The results of the computations are collected in Table III, in which V_0 is the solar velocity; π_{par}, the mean parallax computed from the parallactic motions; θ, the mean value of the residual radial velocity; π'_{pec}, the mean parallax derived from the τ-component; π''_{pec}, the mean parallax computed from μ_1 and μ_2; M, the adopted absolute magnitudes. The values of the parallaxes in the fifth, seventh, and eighth columns correspond to apparent magnitude 5.0. It is evident from the detailed computations that the values of π_{par} are rather uncertain, and deserve less weight than those of π'_{pec} or π''_{pec}. This is partly because of the relatively greater uncertainty in V_0 than in θ.

[1] These equations are analogous to equations (21) and (22) in *Mt. Wilson Contr.*, No. 199; *Astrophysical Journal*, **53**, 13, 1921.

From the figures in the last column of Table III the following estimates of the absolute magnitudes for each spectral class have been made

M2e	−0.6	M6e	+0.5
M3e	−0.6	M7e	+0.6
M4e	−0.5	M8e	+0.6
M5e	−0.0	Se	+0.4

The apparent magnitudes which have been used in all the computations described above are the maximum magnitudes of the mean light-curves of the stars concerned. These necessary data were supplied through the kindness of Dr. Shapley and Mr. Campbell, of the Harvard College Observatory.

CHARACTER OF THE SPECTRUM

The general appearance of the spectrum of a typical long-period variable, as well as the relative intensities of certain lines, is that belonging to a giant rather than a dwarf star. Mr. Adams and Mr. Joy have had the kindness to determine the absolute magnitudes of seventeen long-period variables in the same manner as for ordinary M-type stars. The mean result is −0.5. If we applied the spectroscopic criteria for class M to stars of class S, about the same absolute magnitude would be found for them. This value cannot be given high weight in either case, as it is based on the assumption that the correlations between line-intensities and absolute magnitudes determined for ordinary M-type stars may properly be used without change for variables of classes M and S. It may be said, however, that the spectra indicate pretty definitely that variables of these two classes have, at maximum, luminosities comparable with those of giant M stars. The indications are also that stars of the early subdivisions of class Me are brighter than those of the later subdivisions.

THE DOUBLE STAR X OPHIUCHI

It seems possible that a thorough search might disclose some companion stars to long-period variables, but at present none is known, and this possible means of determining the absolute magnitude is not now available. The lists of double stars contain but a

single object in which one component is a long-period variable.[1] This star, X Ophiuchi, consists of a variable of class M6e, and an invariable component of class Ko, separated by 0".22. The spectra of the two stars can be obtained separately only through the fortunate circumstance that at maximum the variable is much brighter, and at minimum much fainter, than the other component. The absolute magnitude of the Ko star, as determined by Mr. Adams by the usual spectroscopic criteria, is +2.4. From this it follows that the absolute magnitude of the variable at maximum is +0.3. For details of the observations the reader is referred to *Contribution* No. 261; *Astrophysical Journal*, **57**, 251, 1923.

GENERAL DISCUSSION

Numerical values of the absolute magnitude of stars of class M6e derived in this paper by three different methods are as follows:

Method	Spectral Type	Absolute Magnitude
Trigonometric Parallax.....	M6.6e	+0.4
Proper Motion.....	M6e	+0.5
X Ophiuchi.....	M6e	+0.3

The agreement gives us confidence that the proper-motion method, which yields the most extensive data for the various spectral subdivisions, is a reliable one. The mean value for all stars of class Me is +0.1. Wilson has published[2] the value −0.9, but he has informed us that a later unpublished investigation indicates a mean absolute magnitude about 0.0. Gyllenberg's value[3] of −0.6 (reduced to a distance of 10 parsecs), corresponds to about +0.1 for the maximum magnitudes of the *mean* light-curves. We may then adopt +0.1 as the mean absolute magnitude of long-period variables

[1] Since this article was written o Ceti has been found to be a visual double star. See *Harvard College Observatory Bulletin*, No. 792. The companion is of class Bep (Joy, *Popular Astronomy*, **31**, 237, 1923). An independent determination of its absolute magnitude is not possible at present. If the variable component has at maximum the average absolute magnitude of stars of type M6e, +0.5, as determined in this paper, then the absolute magnitude of the companion is +7.

[2] *Astronomical Journal*, **34**, 183, 1923.

[3] *Arkiv för Mätematik, Astronomi och Fysik*, **14**, No. 5, 1918; Lund. *Meddelanden*, No. 90.

of class Me. Table III indicates that variables of class Se have about the same luminosity. Variables of class N are probably brighter.[1]

If we accept the value +0.1 as the absolute magnitude of a typical long-period variable, and make some assumption as to the surface brightness, the size may be calculated. A typical variable at maximum is about one hundred times as bright as the sun. The surface brightness, on the other hand, appears to be approximately one hundredth that of the sun.[2] From these figures the area is 10,000 times that of the sun, and the diameter 100 times the solar diameter. The measurements of total radiation will also yield a result. The absolute bolometric magnitude of a typical variable of class Me at maximum is approximately eight magnitudes greater than that of the sun.[3] Its surface temperature is roughly one-half that of the sun. The energy radiation per unit area, according to Stefan's law, is $(1/2)^4$ as great. The area is then $10^{0.4\times 8}\times 16 = 25{,}000$ times that of the sun, and the diameter 160 times. We may then consider the diameters of variables of class Me to be one or two hundred times that of the sun. The angular diameters will then be equal to or twice the parallaxes. The probable parallaxes of Me stars of apparent magnitude m are given in the second column of the following table:

m	π	Star	m	π
3.0	0".025	o Ceti	3.4	0".020
4.0	0.016	R Hydrae	4.6	0.013
5.0	0.010	χ Cygni	5.3	0.008
6.0	0.006			

For variables with the more advanced spectra, including o Ceti, R Hydrae, and χ Cygni, the diameters may be somewhat larger, possibly two or three times the parallaxes, and perhaps they are within the range of measurement of the interferometer.

[1] Moore, *Lick Observatory Bulletins*, **10**, 167, 1922; R. E. Wilson, *Astronomical Journal*, **34**, 183, 1923.

[2] See Tables XI and XII of *Mt. Wilson Contr.*, No. 226; *Astrophysical Journal*, **55**, 165, 1922.

[3] *Annual Report, Director of Mount Wilson Observatory*, 1922, p. 238.

A variable with a diameter one hundred times that of the sun has a volume one million times as great. If the mean density were the same as the sun's, the mass would be one million times as great; if the mass were the same, the density would be one millionth that of the sun. It is more likely that the density is much less than the sun's than that the mass is much greater. In either event, conditions would be widely different from those which prevail upon the sun, and the true explanation of the spectroscopic peculiarities and the variability in light probably depends on factors which in the sun are absent or of minor consequence.

The stars of the earlier spectral subdivisions of class Me appear to be brighter than the others, as shown by Table III. That they should have higher velocities than the fainter stars is contrary to the usual relationship, and may indicate small mass, but no means of substantiating this supposition is known.

MOUNT WILSON OBSERVATORY
August 1923

EFFECTS OF ATMOSPHERIC DISPERSION UPON DETERMINATIONS OF STELLAR PARALLAX

By FREDERICK SLOCUM

ABSTRACT

Observational test for error in parallax determinations due to star colors.—Systematic errors in stellar parallaxes determined by the trigonometrical method appear to be different for stars of different colors. It has been suggested that this may be due to atmospheric dispersion. To test this a pair of stars, one red, the other blue (spectral types, Mb and B5), were selected, and the field photographed with the 20-inch refractor of the Van Vleck Observatory at hour angles varying from 0^h to $4^h\ 30^m$. Isochromatic plates and a color filter were used.

The results of the test.—The positions of the colored stars were measured with respect to six comparison stars of about the 10th magnitude. From 40 exposures on 20 plates the displacement in zenith distance at altitude 45°, relative to the group of comparison stars, is for the red star $-0''.025 \pm 0''.019$, and for the blue star $+0''.016 \pm 0''.019$. The signs are as would be expected, refraction raising the blue star more and the red star less than the faint comparison stars, probably of about type G. For parallax work, only the components of these values in right ascension need be considered, and these are so small that for plates taken within half an hour of the meridian no appreciable errors due to atmospheric dispersion need be feared.

In a paper presented before Section A of the American Association for the Advancement of Science in 1914, the writer called attention to an evident case of systematic error in determinations of stellar parallax. Since then, as the number of parallax results has increased, many attempts have been made, by the comparison of the parallaxes of different observers, to detect the existence of systematic errors. One interesting outcome of these investigations is that, in some series, the systematic differences seem to vary with the color of the parallax stars. It has been suggested that this may be due to atmospheric dispersion.

In the case of a B-type star, for example, the parallax star will be bluer than the faint comparison stars. Blue rays are refracted more than yellow or red. The right ascension component of this difference will vary both with the zenith distance and with the hour angle. There is a natural tendency to take morning plates at a greater eastern hour angle than evening plates, in order to get larger parallax factors. If this is done, the atmospheric dispersion acts differently upon the morning and evening plates, and this

difference goes directly into the parallax, reducing the positive parallactic displacement as compared to the negative, thus giving for a blue star a result algebraically too small.

Obviously the remedy is to take all plates near the meridian, and the evening and the morning plates at the same hour angle, but this is not always feasible, and the question arises, How much variation is allowable?

There are, of course, many factors involved, but I shall consider only the question of dispersion. The laws of atmospheric dispersion are well known. If we consider the visible spectrum as extending from λ 6800 to λ 4000, and let ΔR be the difference in refraction for the extreme wave-lengths, then according to Kayser and Runge

$$\Delta R = 0.023\ R,$$

that is, the difference in vertical refraction of the red and blue ends of the visible spectrum is 0.023 of the refraction itself. For different zenith distances this gives the values in Table I. At a zenith

TABLE I

z	ΔR
0°	0″.00
10°	.24
20°	.49
30°	.77
40°	1.12
50°	1.60
60°	2.44
70°	3.65

distance of 40°, for example, a star's image would be elongated into a spectrum, the length of which, between the above-mentioned limits, would be 1″.12. This figure, of course, does not apply to the length of a star image on a photographic plate. How much of the spectrum—and what part—is photographed depends upon (1) the color curve of the objective, (2) the color curve of the plate, (3) the color of the filter, if one is used, (4) the color of the star, (5) the magnitude of the star, (6) the length of exposure, (7) the hour angle and zenith distance of the star, and probably other conditions. Theoretically the problem is rather complicated. In an attempt to find out by actual observations the order of magnitude of the

atmospheric dispersion on plates taken with the visual refractor of the Van Vleck Observatory, aperture 20 inches (50.8 cm), focal length $27\frac{1}{2}$ feet (8.38 m), I selected two stars of different colors, B.D.$+47°3321$ and B.D.$+47°3322$, spectral types Mb and B5, visual magnitudes 7.09 and 6.36. These stars are about $2'$ apart. Their positions were referred to six comparison stars of about the tenth magnitude. The diameters of the colored stars were reduced by the rotating sector to approximately the mean of the diameters of the comparison stars. Table II gives data for the stars used. A is B.D.$+47°3321$ and B is B.D.$+47°3322$. Forty exposures on twenty plates were secured at hour angles ranging from 0^h to 4^h. The exposures were all made during October and November, 1922, so there will be no appreciable displacement due to parallax or proper motion. The plates were all reduced to a standard, as in parallax work by the method of dependencies, and then the residuals for the colored stars were investigated for evidences of atmospheric dispersion.

TABLE II

COMPARISON STARS

Star	Diameter	X	Y
	mm		
1	0.12	−15.″2	+ 2.″8
2	.11	− 8.9	− 8.2
3	.08	− 2.6	+ 9.8
4	.07	+ 5.5	+11.0
5	.08	+ 5.6	−13.0
6	.09	+15.6	− 2.4
A	.08	− 1.5	+ 0.4
B	0.08	− 1.1	− 1.8

If we let β represent the coefficient of refraction for the comparison stars and $\beta+\Delta\beta$ the coefficient of refraction for a star whose color differs from the mean of the comparison stars, then $\Delta\beta \tan z$ will be the effect of atmospheric dispersion in zenith distance. The component of this parallel to the equator is

$$\Delta\beta \tan z \sin q = \Delta\beta \tan z \cos\phi \sin t \csc z = \Delta\beta \cos\phi \sin t \sec z,$$

where q = the parallactic angle of the star, t = the hour angle, ϕ = the latitude, and z = the zenith distance.

Let $\sin t \sec z = a$ and $\Delta\beta \cos \phi = x$. then each plate will give an equation of the form

$$ax + y = m,$$

where y is a constant for a particular field, m the residual for the colored star, x the dispersion coefficient multiplied by a constant, and a the dispersion factor varying with the hour angle and the zenith distance. The values of these quantities are given in Table III.

TABLE III

Plate Number	Hour Angle	Weight	Solution m		Dispersion Factor (a)	Residual	
			A	B		A	B
			mm	mm		mm	mm
228.....	0^h36^m	1	+0.0076	−0.0220	0.157	+0.0014	−0.0025
229.....	1 52	1	.0096	− .0230	.502	− .0009	− .0014
230.....	2 23	1	.0100	− .0243	.650	− .0014	+ .0000
231.....	2 50	1	.0084	− .0228	.784	+ .0001	− .0014
232.....	3 25	0.7	.0078	− .0248	.970	+ .0005	+ .0006
233.....	4 06	0.4	.0078	− .0232	1.211	+ .0002	− .0005
234.....	0 20	1	.0074	− .0262	.088	+ .0016	+ .0016
235.....	0 52	1	.0096	− .0260	.230	− .0007	+ .0015
236.....	1 26	0.8	.0090	− .0245	.383	− .0002	+ .0001
238.....	0 12	1	.0103	− .0256	.053	− .0012	+ .0010
239.....	1 22	1	.0089	− .0234	.365	− .0001	− .0010
240.....	1 52	1	.0067	− .0236	.503	+ .0020	− .0008
241.....	2 19	1	.0081	− .0229	.632	+ .0005	− .0014
242.....	2 50	1	.0088	− .0220	.786	− .0003	− .0022
243.....	3 20	1	.0082	− .0248	.944	+ .0002	+ .0007
244.....	3 54	0.9	.0051	− .0236	1.130	+ .0030	− .0004
246.....	0 22	1	.0105	− .0256	.097	− .0014	+ .0010
247.....	1 39	1	.0080	− .0258	.442	+ .0008	+ .0014
248.....	3 15	1	.0111	− .0262	.917	− .0027	+ .0020
249.....	4 39	0.9	+0.0092	−0.0256	1.416	−0.0011	+0.0016

The normal equations from the data in Table III are

		A	B
9.03266 x	+ 10.910 y	= +0.092363,	−0.263992
10.910 x	+ 18.7 y	= +0.16185,	−0.45472

From which, for A

$$x = -0.000774 \text{ mm} \pm 0.00057 \text{ mm}$$
$$= -0''.019 \pm 0''.014$$

and

$$\Delta\beta = -0''.025 \pm 0''.019.$$

For B

$$x = +0.00049 \text{ mm} \pm 0.00058 \text{ mm}$$
$$= +0''.012 \pm 0''.014$$

and

$$\Delta\beta = +0''.016 \pm 0''.019.$$

The scale of the plates is 1 mm = 25″.

Since the values of $\Delta\beta$ are of the order of magnitude of the probable errors, it is evident that the effects of atmospheric dispersion are not appreciably greater than the errors of observation. This is to be expected since the range of wave-lengths registered upon an isochromatic plate after the light has passed through a yellow filter is very small, surely not more than a few hundred angstroms.

The signs of the results are as they should be, the coefficient of dispersion for a red star being less than for the comparison stars, and the coefficient for the blue star greater.

If we assume that the maximum coefficient of dispersion to be expected with the apparatus used is $\Delta\beta = 0''.025$, and translate this into the effect in right ascension for different hour angles and declinations we get the results shown in Table IV.

TABLE IV

DISPERSION IN RIGHT ASCENSION

Declination	Hour Angle		
	30^m	60^m	90^m
−20°	0″.005	0″.011	0″.016
0	.003	.007	.011
+20	.003	.005	.008
+40	.002	.005	.007
+60	0.003	0.005	0.008

From Table IV it is evident that for parallax work with a visual refractor, if the morning and evening plates do not differ in hour angle by more than 30 minutes, no appreciable error due to atmospheric dispersion may be expected, except possibly in the case of stars of extreme color-index, or very low declination. For other reasons it is, of course, desirable to keep the difference of mean hour angles of morning and evening plates well within the limit mentioned.

This subject has been investigated also by Hudson, *Publications of the Allegheny Observatory*, **6**, No. 2, and by Davidson and Greaves in *Monthly Notices of the R.A.S.*, **83**, 56, 1922. Hudson obtained with the Allegheny photographic refractor for stars of type Ma, $\Delta\beta = -0''.093 \pm 0''.004$. Davidson and Greaves give a series of values of $\Delta\beta$ for different spectral types, using the assumed effective wave-lengths for the Greenwich 30-inch reflector. For the comparison stars they have taken λ 4370 and, from the experimental results of Kayser and Runge, have adopted the value $0''.08$ per 100 A as the rate of change of β.

For an M-type star they give $\Delta\beta = -0''.012$ for the reflector, and, for the 26-inch photographic refractor, six-tenths of that amount, or $\Delta\beta = -0''.07$. This agrees well with Hudson's value. Both are very much larger than mine and this is to be expected, partly because the plates used with a photographic refractor are sensitive to a much wider range of wave-length and partly because the prismatic dispersion is much greater in the blue than in the yellow region, to which the isochromatic plates are limited by the use of the yellow filter.

To study the effect of $\Delta\beta$ upon the parallax, the Greenwich observers used a series of parallax plates of a K-type star, all plates presumably taken near the meridian. In the Van Vleck and Allegheny investigations, stars of marked color were photographed at extreme hour angles and no assumptions have been made in regard to effective wave-lengths.

VAN VLECK OBSERVATORY
MIDDLETOWN, CONN.
August 17, 1923

SERIES SPECTRA IN OXYGEN IN THE REGION λ 900–λ 1400

By J. J. HOPFIELD

ABSTRACT

Description of the apparatus employed.—A 50-cm focus vacuum spectrograph and special non-contrast ultra-violet photographic films were used. The mounting was such as to admit five orders of λ 1000 in sharp focus on the films. The discharge tube was H-shape with a total length of one meter, the horizontal portion facing the slit. The gas pressure in the discharge tube and in the receiver was controlled by a capillary needle valve of glass. In addition there was a plate glass condenser, a 6600-volt, 5-kw. transformer, and the necessary pumps.

Discussion of the spectrograms.—New series of triplets of constant frequency difference, due to oxygen, have been found in the extreme ultra-violet. The members of these series are formed according to the combination principle by the transitions of the electron from the previously known levels of oxygen, $1s$, $1S$, $2S$, . . . , $7S$, $2D$, $3D$, . . . , $7D$, to a new triple level OP_{123}. The term values of this triple level are 109607, 109674, and 109833 frequency units respectively. This triple level is closer to the nucleus of the oxygen atom than any previously found for this element and is probably the *valence level*. In this last respect it corroborates the new Bohr theory for oxygen. Since this triple level is the valence level, it suggests the existence of three forms of atomic oxygen, two of these forms being metastable. The *resonance and ionizing potentials* of the stable form of atomic oxygen, as calculated from the data of these series, are 9.106 and 13.56 volts respectively.

INTRODUCTION

Triplet and singlet spectra in oxygen have been known for many years from the pioneer work of Paschen and Runge.[1] The series discovered by them lie in the region between the infra-red and the near ultra-violet. Other unclassified lines of oxygen were found as far as λ 1800.

At λ 1750 a strong absorption band of oxygen sets in and extends to about the limit of fluorite transmission.[2] Oxygen then becomes relatively transparent for a small range of wave-lengths shorter than λ 1300.[3] The new oxygen series recently found by the author lies for the most part in this region of transparency.[4] Besides these spectra, which are best produced by mild excitation, there exists a spark spectrum of oxygen in the extreme ultra-violet. This is described in a later paragraph.

[1] *Annalen der Physik*, **61**, 641, 1897; *Astrophysical Journal*, **8**, 70, 1898.

[2] Lyman, *Spectroscopy of the Extreme Ultra-violet*, p. 67.

[3] Hopfield, *Science*, **54**, 553, 1921.

[4] *Physical Review*, **21**, 710, 1923; *Nature* (London), **112**, 437, 1923.

APPARATUS

A vacuum grating spectrograph of 50-cm focus was used. The mounting is such as to give as many as five orders of λ 1000 in sharp focus on the film. The discharge tube is H-shaped, and is made of 1-cm pyrex tubing. It has a total length of a meter. The horizontal portion faces the slit end-on. The depending legs are bent sharply at right angles at their lower extremities and open into the sides, near the bottoms, of vertical glass cylinders. These cylinders are 2.5 cm in diameter, 20 cm long, and contain the electrodes. Aluminum electrodes, made of the sheet metal rolled into cylindrical form, were used. They fit the cylinders snugly except at the lower ends where the glass is slightly bulged away from the aluminum. This device protects the glass from sparks from the electrodes. Lead-in wires are sealed through the tops of the cylinders and near the leads are the intake tubes for the gas under test. The lower ends of the cylinders, below the side openings mentioned above, serve as wells for aluminum cuttings. These cuttings glow when bombarded by the discharge at low pressures and thus help to maintain the current, and they also protect the walls of the glass from overheating due to the direct bombardment. The electrodes are kept cool by immersing the glass containers in jars of distilled water. The rest of the apparatus consists of the necessary pumps, a 6600-volt, 5-kw. transformer, and a glass plate condenser. The slit of the spectrograph is provided with an iron occulting shutter which is operated by a magnet from the outside. A capillary needle valve of glass and controlled by an electromagnet is used to regulate the gas pressure in the discharge tube and in the receiver.

The electrode farther from the slit is grounded. The glass tube joining the receiver and the pumps is quite long, so that the discharge does not go readily from the other electrode through the receiver to ground. These precautions reduce the fogging of the spectrograms materially. The gas flows from the electrolytic generator through the capillary needle valve into the discharge tube, thence through the slit into the receiver, and finally out into the air by way of the pumps. A McLeod gauge is used to measure the pressure in the receiver and the discharge tube.

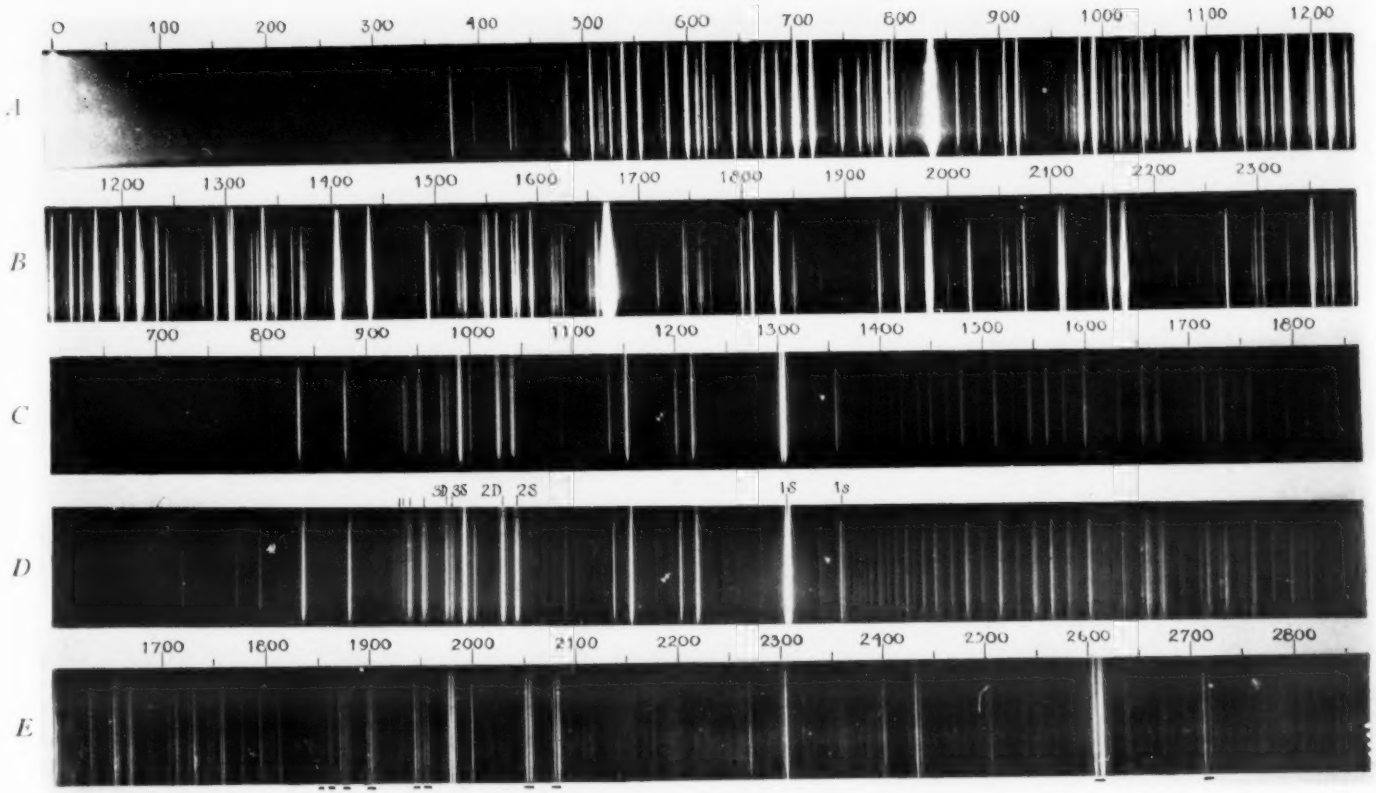
METHOD

In obtaining the spectrograms special non-contrast ultra-violet films were used. The films are allowed to remain in the dry vacuum of the receiver for three or more hours in order that they may shrink to constant length before the pictures are taken. Dry oxygen is then continuously pumped through the instrument. When, under these conditions, the desired vacuum is maintained constant, the occulting shutter is closed in front of the slit, the discharge is started and is allowed to run until the right conditions prevail. The shutter is then opened and the picture taken. Exposures vary in length from a few minutes to many hours.

Some of the visual observations of the spectrum tube may be of interest. At low pressures when the discharge is first started it has a white appearance. Faint bands and a continuous spectrum extend throughout the visible region, and line spectra are scarcely noticeable. After the discharge is run for several minutes the white becomes tinted with pink and the series lines make their appearance. If at this stage a horseshoe magnet is put astride the tube, the discharge at this place becomes white again and once more the continuous and band spectra are prominent in this portion of the tube. The glass in the immediate neighborhood glows with the yellow sodium light. Apparently the atomic ions are more easily deflected by the magnetic field than the molecular ions, and consequently a proportionately greater number of molecules are left in the center of the tube to radiate the band spectra. The walls of the tube glow with the heat developed from the impinging charged particles.

The discharge also changes appearance with pressure. For instance, at a pressure of about 0.3 mm the discharge becomes nearly red and the walls of the tube also appear red as if by fluorescence. An examination of the discharge with a spectroscope shows strong lines on a background that is nearly black. This condition seems to be especially favorable for bringing out the series lines and suppressing the other spectra. Unfortunately, on account of the absorption at this relatively high pressure, it is not the condition best suited to the whole range of the extreme ultra-violet. In all stages of excitation discussed above, the Schuster bands are

PLATE III



SPECTRUM OF OXYGEN

A, *B*, spark spectrum; *C*, *D*, *E*, arc spectrum

best seen in the greenish-yellow glow at the electrodes and are not prominent in the rest of the tube.

DESCRIPTION OF THE PLATE

A and *B*.—Sections of the same spectrogram. Spark spectrum of oxygen. 0.001 mm pressure. Length of exposure, 45 minutes.

C and *D*.—Different prints from the same spectrogram, the one shows wave-lengths and the other series designations. Arc spectrum of oxygen. 0.001 mm pressure. Exposure 16 hours.

E.—Another section of the foregoing spectrogram showing the series lines in the second order.

DISCUSSION OF THE SPECTROGRAMS ON PLATE III

A.—Spectrum lines that appear on the short-λ side of λ 300 are clearly shown on the original negative. A line also occurs at λ 167, and is probably the 1/5 order ghost of the very strong group of lines at λ 834.5. The wave-length scales have been affixed to the pictures with considerable care, and may be read with accuracy to about 1 A. The fourteen lines listed by Millikan[1] as belonging to oxygen, a spectrum also found independently by the author[2] by the use of the ordinary discharge, are clearly shown. Scores of other strong lines also occur. Some are due to nitrogen, others to hydrogen, and still others probably to carbon, and all of these, with the many oxygen lines, make the spectrum very complex. Some of the lines are single and others are not. For example, the following line structure is apparent in the higher orders of some of the spectrograms: λλ 719, 788, 790, and 796 are single; λλ 581 and 616 are double; λλ 507 and 703 are triple; λλ 600 and 685 are quadruple; λ 539 is a quintuplet, etc. This spark spectrum of oxygen has not yet been studied in detail in respect to the relations suggested by the Sommerfeld-Kossel Displacement Law. It is also quite evident that the strong aluminum lines at λλ 1854 and 1862 do not appear on any of the spectrograms, and hence, probably no electrode lines occur with this form of discharge tube. Only the strongest of the series lines of the arc spectrum, as shown by the

[1] *Proceedings of the National Academy of Sciences*, **7**, 289, 1921.

[2] *Physical Review*, **20**, 584, 1922.

A, B, spark spectrum; *C, D, E*, arc spectrum

TABLE I

Series Designations	λ Calculated	λ Observed	I	ν	Δν	Reference to Notes
OP						
1	1359.84	(absent)				*a*
2—1*S*	8.60	1358.66	5	73601	159	
3	5.67	55.73	8	760		
OP						
1	1306.09	1306.12	10	76562	68	
2—1*S*	4.95	4.96	10	630	158	
3	2.25	2.27	10	788		
2*S*	1041.72	1041.71	7	95996	65	
	40.99	1.00	8	6061	161	
	39.27	39.26	8	222		
3*S*	978.64	978.62	4	102184	65	
	.00	.00	5	249	157	
	6.48	6.50	5	406		*h*
4*S*	952.98	952.96	2	104936	66	
	.36	.36	4	5002	156	
	50.92	(50.95)	4	158		*b*
5*S*	939.87					*c*
	.28	939.22	2	106471	155	
	7.88	(7.85)	3	626		*d*
6*S*	932.26					*e*
	1.68	931.75	2	107324	175$_k$	*f*
	0.30	0.24	2	499		
7*S*	927.40					*c*
	6.83	926.91	1	107885	169$_k$	*j*
	5.47	5.46	1	8054		
OP						
1	1028.25	1028.21	7	97256	68	
2—2*D*	7.54	7.49	8	324	157	
3	5.87	5.84	9	481		*g*
3*D*	973.92	973.92	4	102677	70	*h*
	.29	.26	5	745	159	*h*
	1.78	1.76	8	906		*i*
4*D*	950.76	(950.79)		105175	69	*b*
	.16	.17	4	244	160	
	48.73	48.73	4	404		
5*D*	938.65	938.59	1	106542	64	
	.06	(.03)		606	160	*d*
	6.67	6.62	3	766		
6*D*	931.51					*f*
	0.93	930.87	2	107426	148$_k$	
	29.55	29.59	2	574		
7*D*	926.90					*j*
	.32	926.33	1	107952	165$_k$	
	4.96	4.92	1	8117		

NOTES WITH REFERENCE TO TABLE I

a) This line is definitely absent, which is also the case with the corresponding line of the sulphur spectrum (*Nature, loc. cit.*). The great absorption band of oxygen sets is strongly in the neighborhood of the missing line, but at this greatly reduced pressure the absorption band is not noticeable, and consequently the line is not absent because of it.

b) These lines are practically superimposed and in the spectrograms are observed as a single line of increased intensity. The individual lines are then calculated from this observed center of gravity.

c) Not observed. It would be too faint for observation.

d) Superimposed and not observed separately, but "λ observed" is obtained as before from the observed center of gravity of the two lines.

e) Too dim for observation. There remain four lines of the 6*S* and 6*D* triplets and the wave-lengths of these are given.

f) This wave-length is really the center of gravity of two lines, but the 6*D* line is too weak to take into consideration.

g) It is to be noticed that this wave-length is practically coincident with the second line of the hydrogen Lyman series which occurs at λ 1025.73. This fact accounts for its slightly increased intensity. However, this oxygen triplet (2*D*) has been obtained on other spectrograms when the hydrogen lines, even the strong resonance line λ 1215.67, were absent.

h) At slightly higher pressures, these lines are not present, although shorter wave-lengths of the series as well as longer ones occur. The simplest explanation of this fact is that an absorption band which is very steep on the long wave-length side occurs in this region.

i) λ 971.76 is much stronger than would be expected from the law of diminishing intensities for series, and consequently another line of the same wave-length is probably superimposed on the series line at this place.

j) Too dim for observation.

k) These lines are too faint for accurate setting of the comparator. The large departure of the frequency difference from the constant value 159 is thus accounted for.

last three pictures, occur in this form of discharge, and then only as relatively faint lines. *A* and *B* were reproduced from a film which was overexposed for the purpose of bringing out the more feeble lines and consequently much of the fine structure, such as that mentioned above, is not evident from the reproduction.

C, *D*, and *E*.—A new band spectrum probably due to oxygen is shown on *C* and *D* in the region λλ 1300–1700, the region where the great absorption band of oxygen is so evident at higher pressures. In addition a background of feeble bands that show structure exists almost throughout the entire region that was photographed. The chief interest in this arc spectrum, however, lies in the new series. These are marked in the first order on *D* and underscored

on E, where it is hoped that the individual lines may be seen. By using the combination principle these lines form the indicated series and designate the new triple energy level. This level is closer to the nucleus than any yet observed for this element with the exception, of course, of the X-ray levels. The new level is probably the valence level of oxygen. This point is discussed in a later paragraph. The three new series have constant frequency differences, and converge on this triple level.

To determine these levels accurately two of the stronger triplets were used. One of these triplets, $A_{123}-1S$, the wave-lengths of which are λλ 1306.12, 1304.96, 1302.27, has been accurately determined,[1] and the other, $A_{123}-2S$, has been measured with nearly as great accuracy in the higher orders and has the values λλ 1041.71, 1041.00, 1039.26. A_{123} is this new triple term. Its most probable values in frequency units are 109607, 109674, and 109833 respectively, and accordingly the constant frequency differences are 67 and 159. The series show combinations of this term with s, S, and D terms only, all of which are known terms in oxygen, hence from the selective principle A_{123} is either a P_{123} or a p_{123} term. However, these triplets are not like the ordinary ones found in oxygen. The new ones have much coarser spacing, and are inverted in both the intensity and separation of the lines. These considerations and the fact that the Op_1-1s of the $Op_{123}-1s$ triplet is absent, even when the pressure is so low that the absorption bands vanish, seem to indicate that this level is not of the p but rather of the P sequence, and hence is the OP_{123} level of oxygen. This unusual term designation for a triple term, based on the Sommerfeld theory of inner quant numbers, was suggested to the author by Professor R. T. Birge and is discussed fully by him elsewhere.[2] Table I (p. 118) gives the observed and the calculated wave-lengths of the lines, together with their series designations. Wave-lengths are in I.A reduced to vacuum, and were obtained from the second higher and orders of the spectrum. The Fowler notation is used.

The diagram (Fig. 1) represents some of the terms of oxygen. The new triple term is represented on a greatly magnified scale by

[1] Hopfield and Leifson, *Astrophysical Journal*, **58**, 61, 1923.

[2] *Journal of the Optical Society of America*, 8,233, 1924.

the heavy lines. The ordinate of the diagram is proportional to B, the square root of the denominators of the various spectral terms.

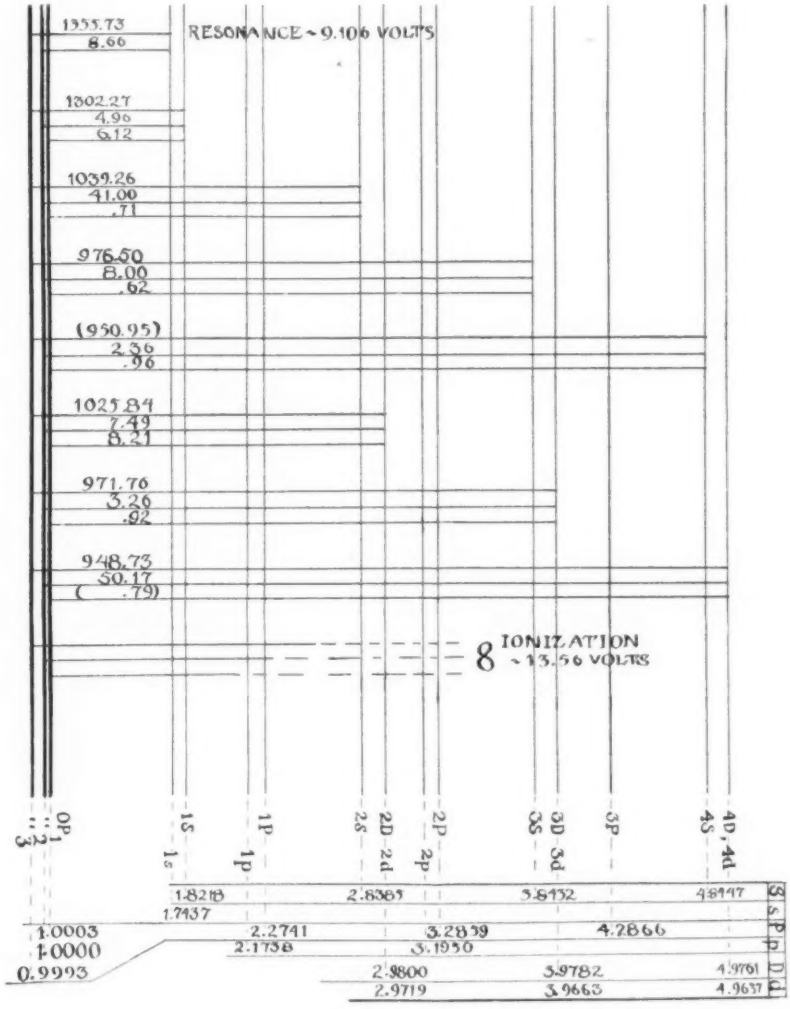


FIG. 1.—Oxygen energy levels

Thus, if the value of the spectral term in frequency units is equal to R/B^2, then $B = \sqrt{R/\text{term}}$, where R is the Rydberg constant. A sample calculation for the 1S term of oxygen follows:

$$B = \sqrt{109677/33043.3} = 1.8218.$$

It is easily seen that, if any of the sequences of terms accurately followed the Rydberg formula, the consecutive B's of the same sequence would have unit differences. The values of a few of the terms of the S, s, P, p, D, and d sequences of oxygen are listed in the vertical columns of the diagram. From these figures it is apparent that none of the sequences follow the Rydberg formula accurately. The S and P terms show a progressive change in one direction, while the D and d sequences vary from the formula in the other direction, while in general the greatest departure is shown by the smallest B's, that is, for energy levels closest to the atomic nucleus. Thus the new triple level whose mean value in the foregoing units is 1.0000 differs from the next higher level of the same sequence by 1.2741. The D and d levels are so nearly coincident that they are represented by the same lines in the drawing. The diagram also shows a list of some of the observed lines and the corresponding electronic transitions. The resonance line and the resonance potential is indicated, and the ionizing potential as calculated from the head of these new series is also given.

DISCUSSION OF DATA

On reviewing the plate it is seen that these series account for most of the lines in the arc spectrum of oxygen in this region. The few lines that remain are λλ 1217.62, 1215.67, a faint and close triplet λ 1199.8, a strong line 1152.0, a triplet 1134.6, a singlet 999.4, and a strong triplet 988.67, 990.13, 990.73 whose $\Delta\nu$'s are 149 and 62 respectively. This last triplet thus has about the $\Delta\nu$ separation of the new series lines, and like them the more refrangible lines are the stronger. Another interesting fact is that a wide space separates the longest wave-length of the new series from the shortest possible wave-length of any previously observed series of oxygen, and no oxygen lines of the arc type appear in this gap. This space represents a span of 37,532 frequency units. Since a complete comparative study with the nitrogen spectrum has not yet been made, one cannot state with certainty the origin of some of these few lines. However, one may note from the plate that the nitrogen pairs at λλ 1742.8, 1745.3, and 1492.6, 1494.6 are not strong; hence the stronger lines listed in this paragraph are probably oxygen

lines. One of the lines, λ 1215.67, is of course the resonance line of hydrogen.

A few other lines which occur in the first order as far as λ 600 are shown on *C*, *D*, and *E*. Some of these on the comparative study of the arc and spark spectra are seen to be intensified in the spark, and, therefore, are spark lines. According to the Sommerfeld-Kossel Displacement Law the spectrum of singly ionized oxygen should resemble the arc spectrum of atomic nitrogen. Unfortunately no line series has yet been discovered in nitrogen, and the study is thus made more difficult. There are, however, some doublets and singlets known for nitrogen, and hence doublets and singlets should exist in the spectrum of singly ionized oxygen. It is seen that some of the lines existing in this region are wide doublets, for example, λ 834 and λ 880 are easily seen as doublets in the third order. On further comparing *A*, *B* with *C*, *D*, *E*, it appears that *A*, *B* contain spectra representative of more than one degree of ionization of oxygen, and this fact probably accounts for their complexity.

A further significance is attached to the new OP_{123} level of oxygen. On the new Bohr theory the valence level of oxygen is indicated as 2_2, which corresponds to a *P* or *p* level. The foregoing series data would corroborate the theory in this detail, but these data also show that this level is a triple one, a fact not predicted by theory. This seems to be the first time that a triple valence level has been found for any element,[1] and a triple valence level entails some interesting consequences. For instance, an application of the Selective Principle indicates that an electron transition cannot ordinarily occur between any two of these orbits, so that the oxygen atom is in the most stable condition when the electron is in the innermost orbit, that is, the one having the highest term (ν 109833), and a metastable atom exists when the electron is situated in either of the other two levels. On the assumption that the foregoing levels are the valence levels, the ionizing potential of the stable form of atomic oxygen is calculated as 13.56 volts, a value practically the same as that of the ionizing potential of atomic hydrogen. The resonance line, that is, the radiation of least energy emitted by the stable

[1] The author has likewise found new series in sulphur which indicate a triple valence level for that element (*Nature*, *loc. cit.*)

form, is λ 1355.73 ($OP_3 - 1s$), and the corresponding resonance potential is 9.106 volts. It may be noted from the spectrograms that this resonance line is not nearly as intense as the $OP_3 - 1S$ line, which may be regarded as a secondary resonance of the atom. This secondary resonance would thus be more prominent in measurements of resonance potential. The resonance and ionizing potentials of oxygen have been experimentally measured by others, but the results of different observers are very different, so their values are not quoted here. If resonance potentials were not limited by definition to the stable form of the atom, the $OP_2 - 1s$ and the $OP_1 - 1S$ lines could also be regarded as resonance lines for the two corresponding metastable forms of oxygen. Similar interpretation would likewise give three ionizing potentials for this element.

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October 1923

THE SPECTRUM OF FLUORINE

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ABSTRACT

The line spectrum of fluorine from λ *7800 to* λ *3400* has been photographed with a 21-foot concave grating, and the wave-lengths measured. Spectra have been taken from a spark between gold electrodes in fluorine at atmospheric pressure and from vacuum tubes at reduced pressures.

The band spectrum of fluorine has been photographed, and the approximate wave-lengths of the heads of ten bands between λ 7000 and λ 5100 have been measured.

Moisson[1] was the first to isolate pure fluorine in considerable quantities. He passed sparks between gold or platinum electrodes in an atmosphere of fluorine at atmospheric pressure, and studied the red lines visually with a three-prism spectroscope. Porlezza[2] used SiF_4 in vacuum tubes, and compared its spectrum with that of $SiCl_4$ and *Si*. He studied the violet end of the spectrum chiefly, photographing the spectrum with a concave grating having 10,000 lines to the inch and a focal length of 3.048 meters. Smythe,[3] at the Ryerson Physical Laboratory in 1921, photographed the spectrum of fluorine with a quartz spectrograph and with a small concave grating, and measured the lines at the red end of the spectrum. One of the plates, taken with an electrodeless discharge at a reduced pressure, showed bands degraded toward the red, and believed by him to be due to CF_4. Most of the spectra were photographed from a spark between gold electrodes in a bulb filled with fluorine at atmospheric pressure.

Ten other investigations have been made since 1862. All, except that of Exner and Haschek[4] in 1912, are cited in Kayser's *Handbuch der Spectroscopie*, Volume V. These have not added materially to our knowledge of the wave-lengths in the fluorine spectrum.

The method used by Smythe for generating the fluorine, and described in his paper, was essentially that developed during the

[1] *Comptes Rendus*, **109**, 937, 1889, and *Annales de Chemie et de Physique* (6), **24**, 224, 1891.

[2] *Gazetta Chimica Italiana*, **42b**, 42, 1912.

[3] *Astrophysical Journal*, **54**, 133, 1921.

[4] *Die Spectrum der Elemente bei Normalen Druck*, **3**, 85.

war by the Chemical Warfare Service. Dry KHF_2 is melted in a copper vessel, and kept at about 245° C. The liquid is electrolyzed by passing a current of a few amperes between the copper vessel, which serves as a cathode, and a rod of pure, hard graphite, the anode. Fluorine has been generated for the present experiment with Smythe's original apparatus, which has been modified in no essential particulars. Gas from the generator is first passed through a tube of fused *NaF* to remove the *HF* which may be present, then through a trap immersed in frozen alcohol at about −100° C. Since *HF* liquifies at 19°.44 C. at atmospheric pressure, any remaining traces of it were condensed and frozen in the alcohol trap, collecting as a white, powdery deposit.

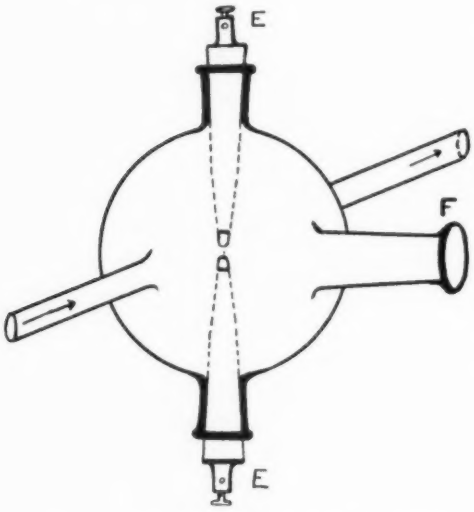
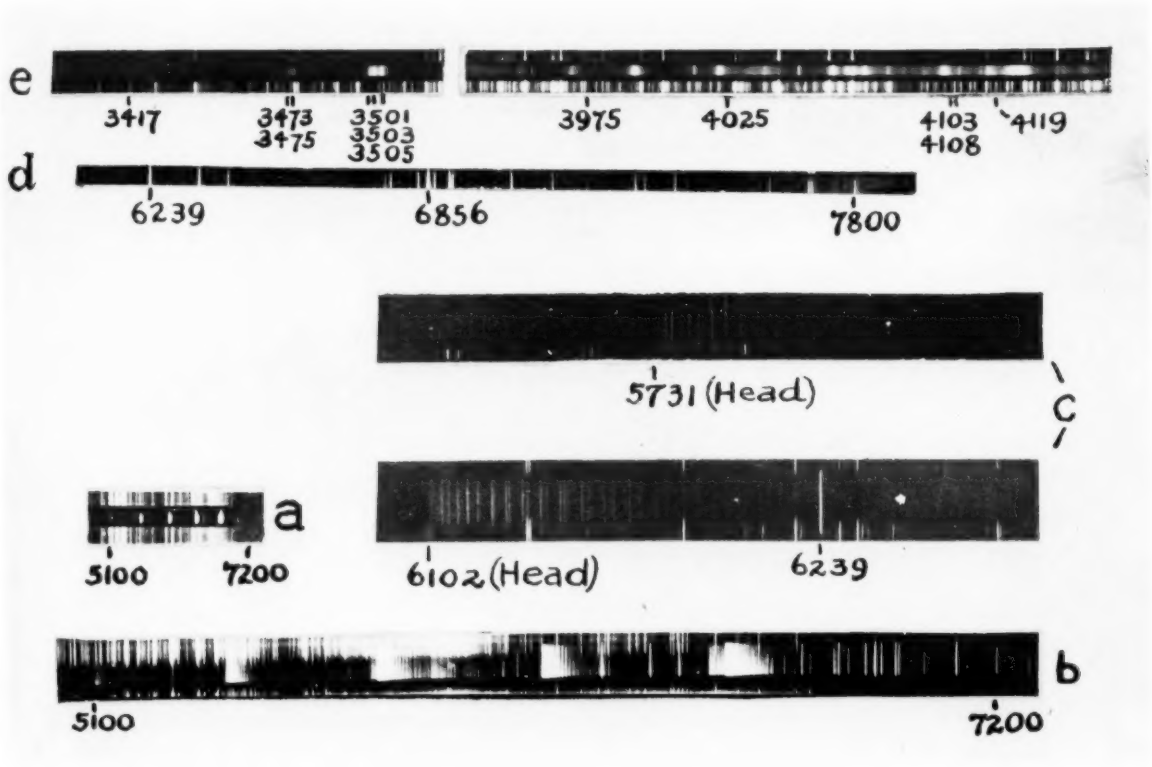


FIG. 1.—*EE*, electrodes; *F*, fluorite window

To obtain the spark spectrum, a steady flow of purified fluorine was passed through a bulb about 2 inches in diameter, fitted with heavy copper electrodes tipped with gold, as outlined in Figure 1. The spark gap was opposite a fluorite window. Pure, cold fluorine does not attack clean, dry glass readily, but the interior of the bulb became gradually coated and opaque after a few days' use. The fluorite window, however, remained clean and perfectly transparent indefinitely. The spark was operated by an induction coil giving a 6-inch spark, used with a condenser in parallel with the spark gap, on a 15-volt storage-battery circuit. The spectra are shown in Plate IV, *d* and *e*. When, after several hours' flow, the gas in the discharge chamber consisted almost entirely of pure fluorine, it became very difficult to operate the spark. For several exposures the storage battery e.m.f. was increased to 25 volts, and

PLATE IV



THE SPECTRA OF FLUORINE

a, band spectrum with quartz spectrograph; *b*, vacuum tube spectrum with 1.5 m concave grating; *c*, bands with 21-ft. concave grating; *d*, lines in red with 1.5 m concave grating; *e*, lines in violet with 21-ft. concave grating.

two induction coils were used, the primaries and secondaries of which were in series separately. Frequently, however, the duration of an exposure was limited by the complete failure of the spark. In the later exposures, one of the spark terminals was made adjustable, so that the length of the spark gap could be altered. Even with this improvement, the length of exposure was limited, as reshaping of the terminals became necessary after three or four hours.

Several times, after the apparatus had been subjected to a heavy flow of fluorine for two or three days, the spectrum of silicon showed in the photographs. These lines were easily identified. It was very difficult to identify the fluorine lines in the blue and violet, where the spectrum of air persisted, no matter how long the fluorine was flowed through the discharge chamber. To determine the fluorine lines, a series of spectrograms was usually taken, the first while the fluorine was displacing the air in the bulb, and a second with the bulb filled with fluorine as pure as possible; then the generator was disconnected and a stream of air blown through the bulb, and another spectrogram was taken when there was no fluorine present.

Photographs were obtained of the spark spectrum in the first order of a 1.5 meter concave grating, and the first order of a 21-foot concave grating. Several times the gas was led first through a discharge tube or spark chamber located in front of the 21-foot instrument, and from there through a second chamber in front of the 1.5 meter spectrograph in the room above. Two photographs were thus obtained simultaneously. If clean when installed, the long length of tubing thus required was not attacked by fluorine, except to a very slight degree after several weeks' use. The spectra obtained by the same form of discharge at different parts of the apparatus were identical, thus indicating that the lines identified as fluorine were probably not due to impurities caused by the action of the fluorine on the glass.

Vacuum-tube spectrum.—Vacuum-tube spectrograms show a series of bands degraded toward the red, between λ 4800 and λ 6550, in addition to the red lines. These bands are shown in Plate IV, *a*, *b*, and *c*. Attempts were first made to obtain the vacuum-tube spectrum photographically by collecting in traps, surrounded by

liquid air, a large amount of condensed fluorine, a clear liquid of a golden hue. To obtain as much as 25 cc of this liquid required continuous operation of the generator for ten or fifteen hours. The trap was then sealed off from the generator. The irrevocable nature of this step and the deterioration of the tubing about the seal caused it to be abandoned. Stopcocks of special design were then employed, but the fluorine attacked the stopcock grease so vigorously that no stopcock could be made to remain tight and not stick, even if it withstood the heat generated. Sufficient fluorine could be collected in this way for a visual examination of the spectrum, but not enough for a prolonged exposure. Finally, a constant flow was obtained, at a pressure of about 5 mm in the discharge tube, by the insertion of a finely drawn capillary between the generator and the discharge tube. Additional regulation of the flow was obtained by means of a large stopcock on the low-pressure side of the capillary. Traps surrounded by frozen alcohol were at first placed on each side of the discharge tube, although later this was found to be unnecessary. A Hyvac oil pump was used to obtain the low pressure. The arrangement is shown in Figure 2.

Smythe first obtained this band spectrum with a discharge tube of the electrodeless type, but attributed the bands to CF_4 due to the backing up of this gas from a carbon tube in front of the pump. In order to determine the origin of the bands, electrodeless tubes and tubes with electrodes of aluminum, iron, copper, and tungsten were used, and the position of the tube in the apparatus was varied. In all cases the bands were the same. The same bands were observed to be very brilliant when a trap full of liquid fluorine was allowed to evaporate quickly into the discharge tube, and they died away as soon as the last of the clear, golden liquid evaporated and was pumped off. In the bottom of such a trap there was usually a white, powdery deposit, which was evidently principally hydrofluoric acid in the solid state. *HF* has a melting-point of $-92°.3$ C., and a boiling-point of $19°.44$ C., which indicates that these bands are probably not due to *HF*, as they disappeared before the white deposit began to evaporate. In order to check this point further, a generator was built for the production of *HF*, and the spectrum observed visually with discharge tubes of the same dimensions as

those used with fluorine, and with the electrical conditions the same. The spectrum observed was the same as seen in the fluorine apparatus after the golden liquid had all disappeared and the white deposit began to evaporate. Attempts to photograph this spectrum were unsuccessful because of the breakdown of the apparatus in a short time under the attacks of the *HF* on the glass. Since the main object had been attained, i.e., identification of the *HF* spectrum as that obtained from the white deposit, and the removal of any possibility that the bands were due to *HF*, nothing further was done along this line.

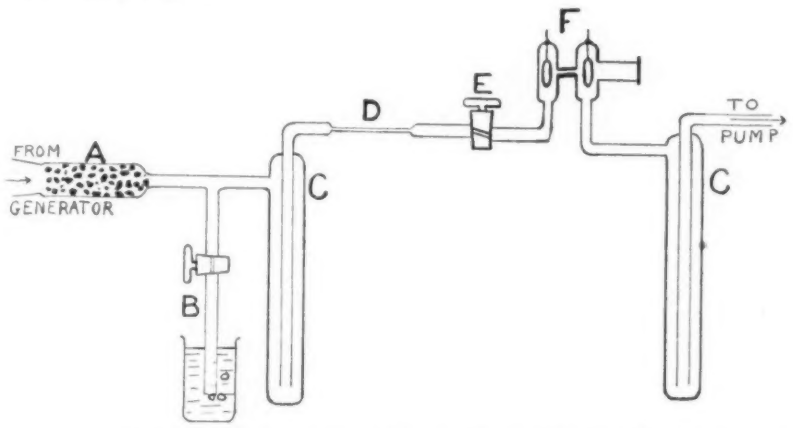


FIG. 2.—*A*, *NaF* tube; *B*, pressure indicator; *CC*, traps in frozen alcohol; *D*, capillary to control flow; *E*, stopcock to control flow; *F*, discharge tube.

It may also be mentioned that it was easy to collect the golden liquid in traps immersed in fresh liquid air (temperature $-192°$ C.), and difficult with old, standing liquid air from which most of the nitrogen (temperature $-195°$ C.) had escaped, leaving a preponderance of oxygen ($-182°5$C.). The boiling-point of fluorine is $-187°$C.

The bands in question are not due to SiF_4. The bands due to this compound were photographed by Mr. Durbin, and formed a beautiful group at entirely different positions in the spectrum. Attempts to secure the spectrum of CF_4 by Mr. Porter and Mr. Blackburn have so far been unsuccessful. Certain new bands have been observed visually by Mr. Blackburn, but they do not coincide with the bands in question.

Line spectra of fluorine.—The lines of the fluorine spectrum fall under two heads. The sharp lines are listed in Table I. These occur both in the spark discharge at atmospheric pressure and the vacuum-tube discharge at low pressures. They are sharper at reduced pressures than at atmospheric pressure, but no changes in their positions or relative intensities have been observed. It will be seen that all the lines of this group are in the orange and red.

TABLE I

LINES OF FLUORINE

(Obtained in Discharge Tube at Pressure of 3 mm Mercury)

Wave-Length	Int.	Wave-Length	Int.	Wave-Length	Int.
5659.02	∞	6239.63*	30	6856.01*	40
5667.48	0	6254.68	2	6870.21*	18
5671.62	1	6348.46*	28	6902.46*	30
5700.72	2	6356.87	1	6909.79*	20
5707.23	3	6405.15	3	6966.33*	4
5967.30	0	6413.61*	23	7037.48*	50
5996.76	1	6416.30	3	7202.34*	15
6014.08	0	6502.04	1	7311.40†	15
6015.90	1	6522.67	1	7332.25†	18
6017.90	2	6569.66*	10	7398.96†	20
6024.02	0	6580.36	4	7426.2†	12
6038.04	1	6650.40	3	7482.95†	5
6047.55	4	6690.48*	5	7552.20†	5
6049.66	0	6762.91	6	7573.32†	5
6039.02	0	6766.98	3	7754.8†	5
6080.13	1	6773.96*	20	7800.5†	2
6149.77	2	6795.47	3		
6210.85	3	6834.25*	18		

* Lines measured also in spark at atmospheric pressure.

† Lines measured only in spark at atmospheric pressure, but of the same general character as other lines in table.

In Table II are listed the broad, hazy lines. These have been observed only in the spark discharge at atmospheric pressure. Unsuccessful attempts were made to obtain them with a vacuum tube of the usual form having a capillary of about 1 millimeter diameter. The tube was operated by an induction coil with a Wehnelt interrupter, carrying about 65 volts on the primary, and with a condenser in parallel with the discharge tube. While the color of the tube operated with an ordinary induction coil and no capacity in the secondary was a brilliant orange, due to the presence of the bands, the tube with the more powerful discharge was

greenish white in the capillary, where no bands were observed, but remained brilliant orange in the wider portions of the tube where the bands persisted. On account of the violent discharge, the glass in the capillary was more strongly attacked, and the silicon lines appeared as soon as the tube was operated.

TABLE II

LINES OF FLUORINE

(Obtained with Spark Discharge at Atmospheric Pressure. Lines not Observed in Discharge Tube)

Wave-Length	Int.	Wave-Length	Int.	Wave-Length	Int.
3417.85	1	3601.70	1	4024.83‖	2
3473.53	1	3603.32	1	4025.09?	1
3475.24	1	3704.75	1	4103.41	2
3501.68	1	3847.05	4	4109.10	1
3503.32	2	3849.97	3	4116.50¶	1
3505.85	3	3851.73	3	4119.35	1
3541.85?	1	3898.80	2	4246.41	4
3590.49*	1	3902.00	2	4299.03	2
3598.16?	0	3905.67?†	1	4361.35	2
3598.98	1	3974.80‡§	1	6762.13	4

Also possible lines at 4368.36, 4398.4, 4400.6; and at 3641.3, 3642.3, 3642.9.
* *Si* 3590.77.
† *Si* 3905.37; *O* 3907.5.
‡ *O* 3973.5.
§ On one plate measured as double: 3972.75, 3976.24.
‖ On one plate measured as double: 4024.80, 4025.65.
¶ *Si* 4116.15.
Doubtful if *Si* lines have any effect, as stronger lines of *Si* did not appear on many of these plates.

Band spectrum of fluorine.—The bands attributed to fluorine for the reasons already mentioned are of fairly equal intensity visually, although the bands of longer wave-length are perhaps stronger. The approximate wave-lengths of the heads of the bands

TABLE III

1	6977.0	6	5851.3
2	6927.2	7	5731.4
3	6518.6	8	5515.5
4	6488.7	9	5393.9
5	6102.6	10	5102.2

are given in Table III. The original of Plate IV, *b*, is on a panchromatic plate, the weakness of which in the green makes the bands of that wave-length seem to be unduly faint. An inspection of the

large-dispersion photographs of the bands, Plate IV, *c*, suggests at once that many of their lines lie on the well-known parabolic curves into which band spectra can be analyzed. Preliminary calculations show that four of these bands yield such branches for about twenty terms, which plot on smooth curves, presumably parabolas since the second differences of the frequencies are approximately constant. The most obvious characteristics of these bands are:

1. They are degraded toward the red.

2. The sharpness of their lines is greatest in the band at the violet end of the spectrum. Each succeeding band toward the red has broader lines, suggesting that the component lines are complex.

3. Each band is composed of a large number of branches. There are two branches of nearly equal intensity dominating each band. In the reddest band photographed with large dispersion, the first two components of these strong branches are double lines.

While they possess many easily recognizable characteristics, such as have been mentioned, these bands are not simple. The work of making a more complete analysis and explanation of their structure is in progress, and will be published later.

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January 1924